Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

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Details of the STSM Title: Learning for Automated Theorem Proving Based on Proof Structures in the Presence of a Large Proof Library Start and end date: 01/04/2024 to 07/04/2024
Goals of the STSM Purpose and summary of the STSM. We have a successful collaboration with Christoph Wernhard on applying learning to enhance proof search for so-called condensed detachment problems: first-order problems that permit particularly simple and versatile proof structures. Our aim is now to extend the scope of this previous work to applications in mathematical knowledge representation with Metamath, a successful large-scale formalisation of mathematics, which is based on condensed detachment. The proposed research visit has the purpose to set out the details of the envisaged further collaboration on this aim.
Working Plan One topic is discussing and elaborating on technical ways to understand proofs in Metamath, which is based on condensed detachment [Megill 1995]. In the context of ATP, the precise understanding of Metamath raises several issues that are not yet settled, although some of them were recently addressed [CBU 2023]. A second topic is to design a family of ATP methods for Metamath that are based on structure enumeration (which grew out of the connection method), similar to the SGCD and CCS provers [W 2022, W 2023, RWZB 2023], that were designed for condensed detachment problems. A particular question of interest is the relationship between proof compression techniques (used during proof search or on given proofs) and some specific techniques used in the human-made formal Metamath proofs.

1 This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

The third topic concerns the possibilities to incorporate learning, similar to what has been done for condensed detachment problems in [RWZB 2023]. We expect that Metamath permits to reuse proof structures as central data, although in a generalised form. Metamath provides much more potential training data than what we had available, e.g. the 40,000 theorems of the "set.mm" Metamath knowledge base together with proofs by humans. The fact that the available proofs are by humans and that the theorems in the knowledge base were selected by humans adds an aspect to learning that was not available in our earlier work: we may take the proofs as instances of "good" proofs and the theorems as instances of "mathematically relevant" theorems.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

We present the expected outputs ordered according to the research coordination objectives of the Action that are immediately addressed by them.

- "Promote the output of detailed, checkable proofs from automated theorem provers."

We work on the interplay of Metamath with ATP. Inherently, proofs emitted by ATP systems have to be detailed and checkable to be of use for Metamath. Already in our previous work [RWZB 2023] we considered ATP with such proofs, technically represented as so-called D-terms. We expect that this approach can transferred to Metamath as it is built on very similar foundations [Megill 1995].

- "Develop the use of artificial intelligence and machine learning techniques on proofs."

Our previous work [RWZB 2023] already considered machine learning, but for a corpus of a few hundred condensed detachment problems. Metamath currently provides about 40,000 proving problems. In addition, its theorems are of mathematical relevance and its proofs are created by humans. This provides an entirely new basis of underlying data as well as utility measures for learning.

- "Provide tools for searching large libraries of formal proofs"

Metamath is a library of formal proofs. A key aspect of ``searching'' is the involvement of semantics, realised through the involvement of ATP systems. Our work on the interplay of ATP and Metamath is expected to be directly usable for searching in this sense. We expect that key techniques of [RWZB 2023] can be transferred from pure condensed detachment problems to Metamath.

The proposed STSM involves the meeting of two researchers who successfully collaborated in the past to set-up and discuss details of a planned further collaboration. Hence, it immediately matches with the Action's capability building objective "Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the Action."

References

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