Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Matteo Spadetto

Details of the STSM

Title: Categorical semantics for propositional dependent type theories

Start and end date: 09/04/2023 to 30/04/2023 (I plan to have a research visit period of about 20 days – of course if it is not possible to be funded for 20 days, I can shorten the visit period to two weeks).

Goals of the STSM

In recent years, there has been a growing interest in various weakenings for theories of dependent types (DTT). In detail, authors working in this field are particularly interested in the weakenings of a DTT with respect to the strength of the computation rules (e.g. whether they hold judgementally or propositionally) for the type constructors of the DTT itself, and in developing new methods and principles for studying these topics from a categorical perspective(see[1],[2],[3]).

During this STSM, I will be working with Benno van den Berg (host), Daniël Otten, et al. to investigate these subjects. Specifically, our research objectives will include exploring general methods and principles for the study of the categorical semantics of both propositional and strict DTTs, and providing a meaningful notion of semantics that unifies the ones appearing in the literature. This will involve working with the established notions of semantics provided by categories with attributes/comprehension categories/etc., as well as with the recent notion of path categories introduced by van den Berg (see[2]). We will investigate how path categories provide a semantics for propositional type theories, and compare this with the previous notions of semantics.

This STSM will bring together researchers interested in weakenings of DTTs and the notion of conservativity. This includes researchers such as Rafaël Bocquet and Théo Winterhalter, whom Benno and Daniël plan to invite to Amsterdam for further discussions and workshops. Additionally, during my time in the Netherlands, I will meet with Benedikt Ahrens and Paige North, with whom I have previously discussed my research on conservativity during the WG6 meeting on syntax and semantics of type theory 20-21 May 2022, for which I was supported by an E-COST-MEETING-CA20111 funding (as I acknowledge in my preprint – see[4]). It is important to foster collaboration and exchange ideas to make progress in this field, and the STSM provides a unique opportunity to do so.

Working Plan

The proposed research project aims to investigate the semantics of DTTs, with a particular focus on the comparison of weak and strong DTTs. The project is expected to contribute to EuroProofNet's Working Group 6 (Type Theory) by aligning with the group's aim of developing a comprehensive theory of type theories.

To achieve this goal, we have the following working plan:

(1) Review and clarify the notion of semantics (which still needs to be made more precise) for propositional type theories (i.e. DTTs with propositional computation rules) provided by the notion of path categories (see[2]), and compare it to the notion(s) of semantics for weak and strong type theories provided by the notion(s) of category with attributes (or category with families, or comprehension category or natural model). In particular, we will investigate the categories of the models of a given weak theory according to both the notions of semantics, and search for a way of converting the ones into the others. Finally, we aim to categorically characterise this construction in terms of (2-/bi-) adjunctions/equivalences.

(2) Investigate what notion of DTT substitution is modelled in the semantics of propositional type theories provided by path categories. This will involve seeking a coherence theorem for path categories that repairs the issue of the substitution, when this is not strictly functorial. The problem is similar to the one for locally cartesian closed categories, for which Martin Hofmann produced a coherence theorem, and our approach will build on Hofmann's argument (see[5]).

(3) Propose a unified notion of category theory-based semantics for any (generalised form of) dependent type theory. This will involve building on the insights gained from the comparison of the different notions semantics for weak and strong type theories, and investigating the nature of the substitution for the unified semantics.

(4) Use the proposed unified notion of semantics to compare the models of a weak theory to the ones of the corresponding strictified theory, with the goal of providing a better description of the notion of "weakening" for the rules of a theory.

(5) Investigate conservativity results for propositional type theories and/or reinforce or generalise the ones of my preprint to appear in arxiv (see[4]). This will involve collaboration with Daniël Otten, a PhD student of van den Berg who works in this topic, as well as with Rafaël Bocquet and Théo Winterhalter, who work on conservativity problems and on propositional equalities as well, and with whom we plan to organize a workshop.

The proposed working plan will be carried out over a longer period than the one of the STSM itself, and will involve regular communication with van den Berg, Otten, and the other experts mentioned above. The project's progress will be evaluated through regular reports and meetings with EuroProofNet's WG6.

Expected outputs and contribution to the Action MoU objectives and deliverables.

The EuroProofNet outlines several objectives and deliverables aimed at developing a comprehensive theory of DTTs and promoting collaboration among researchers working in proof theory. Our research project aligns with these objectives and hence contributes to the overall goals of EuroProofNet, in particular with respect to the ones of WG6 (Type Theory).

The project's objectives include:

(1) Comparing the notion of semantics for weak dependent type theories provided by the notion of path category to the notion(s) of semantics for weak and strong type theories provided by the notion(s) of category with attributes, category with families, comprehension category, or natural model. This will be accomplished by examining the categories of the models of a given weak theory according to both notions of semantics.

(2) Proposing a unified notion of category theory-based semantics for any dependent type theory. This will be used to compare the models of a weak theory to the ones of the corresponding strong theory.

(3) Investigating conservativity results between dependent type theories, including several researchers mentioned above.

In addition to these objectives, the project will also address some additional issues discussed with Benno van den Berg and Daniël Otten. Firstly, we aim to clarify the notion of semantics for propositional type theories provided by path categories. We will examine the problem of the DTT substitution in general, which is not modelled in a strict form by a general path category (as it happens for a general lcc category). We aim to find a coherence theorem for path categories that repairs this issue.

Secondly, we plan to organize a workshop in Amsterdam, bringing together researchers (mentioned above) working on weakened notions of DTT and conservativity problems.

Summarising, this research project aims to contribute to the development of a comprehensive theory of type theories and investigate conservativity results for weakened type theories. Additionally, we will clarify the notion of semantics for propositional type theories provided by path categories and organize a workshop to share ideas with other experts in the field.

[1] Marc Bezem, Thierry Coquand, Simon Huber, A model of type theory in cubical sets, 2014

[2] Benno van den Berg, Path categories and propositional identity types, 2018.

[3] Rafaël Bocquet, Coherence of strict equalities in dependent type theories, 2020.

[4] Matteo Spadetto, A conservativity result for propositional type theories, about to appear in arxiv.

[5] Martin Hofmann, On the interpretation of type theory in locally cartesian closed categories, 2005.