

## Short-Term Scientific Mission Grant - APPLICATION FORM<sup>1</sup> -

Action number:

Applicant name: Anton Setzer

### **Details of the STSM**

Title: **Formalisation of Meaning Explanations in Agda**

Start and end date: 23/08/2022 to 06/09/2022

### **Goals of the STSM**

Purpose and summary of the STSM.

In his paper “Constructive Mathematics and Computer Programming” (1982) and book “Intuitionistic Type Theory” (1984) Martin-Löf has provided so called “Meaning Explanations” for Intuitionistic Type Theory in natural language. They provide a natural language formalisation of the semantics of type theory, and are meant as a philosophical justification of the consistency of type theory. The goal of this project is to formalise meaning explanations mathematically precisely in the type theoretic interactive theorem prover Agda. This provides a road to formalising type theory inside type theory (reflection) and to formalise strong extensions of Martin-Löf Type theory. One example is extending type theory with the extended predicative Mahlo Universe developed by Setzer and Kahle. Setzer gave a preliminary formalisation of this in his talk at the GÖDEL’2021 conference in Tuebingen, Germany. Setzer and Dybjer have carried out preliminary work on formalising the meaning explanations in Agda starting with carrying this out for System T. The goal is to extend this work towards complete type theory and to investigate which principles need to be added to type theory in order to be able to formalise the extended predicative Mahlo Universe or even stronger theories in this way.

### **Working Plan**

Description of the work to be carried out by the applicant.

*(max.500 word)*

<sup>1</sup> This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

- 1) Completion of the formalisation of system T in Type theory.
- 2) Extension to basic dependent type theory with Pi types and natural numbers.
- 3) Formalisation of standard data types of type theory.
- 4) Formalisation of universes.
- 5) Extension to full inductive-recursive definitions.
- 6) Formalisation of the extended predicative Mahlo Universe in Type Theory which for proof theoretical reasons requires termination principles which go beyond Agda.
- 7) Theoretical development of new principles which enable termination checking the extended version of type theory.
- 8) More speculatively: what is the Martin-Löf style meaning of Cubical Type Theory? We would like to throw light on two possible approaches:
  - 9) (i) through a reduction of Cubical Type Theory to the extensional version of Martin-Löf type theory, which itself is justified by the meaning explanations
  - 10)(ii) direct meaning explanations of Cubical Type Theory including the Glue Type used for formalising the univalence axiom.

### **Expected outputs and contribution to the Action MoU objectives and deliverables.**

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

*(max.500 words)*

Contributions to the progress towards Action objectives:

1. A library for formalising meaning explanations in the theorem prover Agda, available as a git repository.
2. The development of a more fundamental level for formalising theories (which allows to formalise such concepts as terms, reduction rules, general typing rules), which can be used for formalising other proof calculi as well and serve as a layer for the interoperability of proof systems.
3. Formalisation of new principles which go beyond induction-recursion and allow to formalise the semantics of theories such as the extended predicative Mahlo universe, and allow the interpretation of stronger proof systems into the framework of Martin-Löf Type Theory.
4. Improved linkage between the Unit of Logic and Types at Chalmers and the University of Göteborg and the Theoretical Computer Science Research Group in Swansea esp in their activities based on the theorem prover Agda.

Deliverables:

1. Agda Library as mentioned before.
2. Several publications in high quality journals and conferences in which the

formalisations are presented.