EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Martina Seidl

Details of the STSM

Title: Checking proofs from QBF solvers in Dedukti and Lambdapi

Start and end date: 19/02/2024 to 23/02/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

During this research we investigated how Dedukti/Lambdapi could be used to check proofs of solvers for quantified Boolean formulas. To this end, we considered three different proof systems

(1) Q-Resolution (the proof system of QCDCL-based solvers)
(2) Forall-Exp Res (the proof system of expansion-based solvers)
(3) QRAT (which subsumes the other two solvers

Q-Resolution and QRAT are extensions of the propositional proof systems resolution and RAT for which formalizations are available. We studied the propositional RAT encoding for Dedukti as well as the first-order resolution encoding. Therefore, we generated several benchmarks for first experiments.

Furthermore, we investigated how the translation of QBF to first-order formulas could be used in order to apply the theorem prover iProverModulo to obtain Dedukti proofs for QBFs. We identified some benchmarks and analysed the proofs in detail.

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

During this short-term mission we worked on the tasks described in the following.

1. Introduction to QBF proof systems and Deducti/Lampdapi

The first task was an in-depth introduction on the various QBF proof systems as produced by recent QBF solvers. We focused on the three proof systems introduced above. Furthermore, also an introduction on Dedukti/Lampdapi and several expressive examples was given.

2. Architecture of the checking process with Dedukti/Lambdapi (DK/LP)

In this task, we evaluated the possibilities to use DK/LP to check QBF proofs. In particular, we identified Q-resolution and QRAT as interesting candidates for further work. On the one hand, there is the tool lrat2dk developed by G. Burel that automatically translates RAT proofs to Dedukti. We experimented with some small formulas that contain only existential quantification and got first insights into what the translation for QBF could look like. For false QBFs, a direct adoption seems possible by expanding universal variables.

The encoding implemented in lrat2dk is based on the simulation of RAT by extended resolution which can also be lifted to QBF. We also considered true formulas for which QBF solvers also can emit proofs. To handle true formulas as well, novel insights are necessary. In order to get resolution proofs, we took a detour via EPR, a subset of first-order logic. In this way, we could exploit the formalization of resolution that is implemented in an extension of the theorem prover iProver which is called iProverModulo. During the visit we could evaluate simple encodings.

3. Suitable benchmarks and first experiments

We crafted several small, but interesting formulas for which we could obtain proofs from the QBF tools and which helped to understand the challenges that need to be overcome in the future.

4. Identification of next steps

As next steps, we identified the following tasks:

- Develop QLRAT for QBFs. LRAT extends RAT by hints that makes the checking much easier and more efficient. Currently, there is no version of QRAT that provides hints.
- Develop methods to deal with quantifier orderings.
- Identify interesting benchmarks from practical QBF encodings.
- In addition, we plan a collaboration on formalizing testing for QBF encodings.