Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111 (EuroProofNet)

Applicant name: Philip Saville

Details of the STSM

Title: Towards 2-dimensional 2nd order algebraic theories

Start and end date: 10/03/2024 to 17/03/2024

Goals of the STSM

Purpose and summary of the STSM.

This project is about building a category-theoretic framework that brings together second-order algebraic theories and rewriting.

Traditional algebraic theories, as studied in universal algebra, describe n-ary operations with no binding. Second-order algebraic theories, introduced by Fiore and collaborators as part of the 'algebraic type theory' programme (e.g. [1,2,3,4]), allow operators with binding. This theory has been influential, but one can only describe equations -- not rewrites -- between terms.

On the other hand, in recent years there has been significant interest in using higher categories to model ways of transforming programs (e.g. [4,5,6]). A category has objects and morphisms, which naturally correspond to types and typed terms-in-context. A 2-category has objects, morphims, and "2-cells" between the morphisms: these naturally correspond to types, terms-in-context, and rewrites between terms.

Our aim is to use 2-categorical techniques to define 2-dimensional versions of 2nd-order algebraic theories and 2nd-order presentations, and develop their fundamental theory. The eventual goal is to provide a modular, universal-algebra style, mathematical framework for describing type theories, together with their binding operators and rewriting rules.

[1] Marcelo Fiore, Gordon Plotkin, and Daniele Turi. Abstract syntax and variable binding. In Proceedings. 14th Symposium on Logic in Computer Science (Cat. No. PR00158), pages 193–202. IEEE, 1999. doi:10.1109/LICS.1999.782615

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

[2] Marcelo Fiore and Chung-Kil Hur. Second-order equational logic. In Computer Science Logic,pages 320–335. Springer, 2010. doi:10.1007/978-3-642-15205-4_26.
[3] M. Fiore. Algebraic foundations for type theories. 18th Types for Proofs and Programs workshop, September 2011. Slides available at https://www.cl.cam.ac.uk/~mpf23/talks/Types2011.pdf
[4] Arkor, N., & McDermott, D. (2021). Abstract Clones for Abstract Syntax. In 6th International Conference on Formal Structures for Computation and Deduction (FSCD 2021). Schloss Dagstuhl-Leibniz-Zentrum für Informatik.
[5] D. R. Licata and R. Harper. 2-dimensional directed type theory. Electronic Notes in Theoretical Computer Science, 276:263–289, 2011. Twenty-seventh Conference on the Mathematical Foundations of Programming Semantics (MFPS XXVII)
[6] M. Fiore and P. Saville, "A type theory for cartesian closed bicategories (Extended Abstract)," 2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), Vancouver, BC, Canada, 2019, pp. 1-13, doi: 10.1109/LICS.2019.8785708.
[7] Ahrens, B., North, P., & Van der Weide, N. (2023). Bicategorical type theory: Semantics and syntax. Mathematical Structures in Computer Science, 1-45. doi:10.1017/S0960129523000312

Working Plan

Description of the work to be carried out by the applicant.

Practical aspects

The visitor (Philip Saville) and host (Dylan McDermott) will work together with Nathanel Arkor (also applying for an STSM). This brings together expertise in each of the strands in the proposal: Arkor and McDermott have already collaborated on categorical universal algebra and Fiore-style second-order algebraic theories (see [4,5] above), and Saville is an expert on 2-dimensional semantics (see [6] above).

At the moment our work is at an early stage. Our plan is to use the time provided by the STSM to develop the basic theory and understand the key definitions and theorems we need. Being in the same room and able to collaborate at a whiteboard for an extended period of time will be invaluable. Once we understand the key ideas, we will be able to divide the work and each focus on different aspects towards an eventual paper submission.

Technical aspects

Our aim is to use 2-category theory to develop 2-dimensional versions of second-order algebraic theories and their presentations.

First, we will give a definition of 2-equational presentations, which one might think of as a specification of a type theory with its binding operators and rewrite rules.

Next we will show this definition is correct by proving theorems paralleling those which hold in the 1-dimensional setting. For example, it is known that the correspondence between theories, presentations, and monads in universal algebra extends to the 2nd-order setting (ie. with the addition of binding). From a type-theoretic perspective, this says that we have an equivalence between ways of describing type theories via their signatures (presentations), the

syntax of type theories (theories), and certain monads. We will show versions of these results in the presence of rewriting.

Finally, if time permits, we will look into other forms of enrichment. 2-dimensional categories are categories enriched in Cat, but it is plausible that for other bases of enrichment a similar development is possible. For example, enriching in suitable categories of topological spaces or metric spaces may shed light on type theories with notions of distance or relative cost, rather than rewrites, between terms. Here the 2-categorical perspective will be useful: it will force us to use formal category-theoretic arguments, which are often amenable to generalisation or being ported to new examples.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

We anticipate that our work will fall squarely within Deliverable D4 ("Definition of a mathematical framework for modular reasoning about type theories and their extensions") and Research Coordination Objective 7 ("Develop a modular theory of type theories").

The work in this STSM to be the starting point for not just this particular project but also for an on-going collaboration focussed on developing category-theoretic techniques for the description of type theories and programming languages. So the STSM will also help achieve the following Capacity Building Objectives:

1. Bring together members of the different communities working on proofs in Europe.
3. Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the Action.
5. Actively support young researchers, the under-represented gender, and teams from regions with less capacity.
7. Prepare competitive EU researchers for a fruitful career in an international environment through intensive use of Short Term Scientific Missions (STSM) and joint educational programs with industry.

Finally, our aim will be to publish this work in a relevant computer science conference (e.g. ETAPS, LICS). The abstract category-theoretic developments may be of significance in their own right, in which case these will be published in a mathematics journal. Together, this contributed to Capacity Building Objective 8 ("Disseminate the results of the Action activities to the scientific community, the industry, the certification bodies, the European institutions and to the general public").