Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Morgan ROGERS

Details of the STSM

Title: Understanding interpretations of HoTT in elementary (oo,1)-toposes

Start and end date: 22/05/2023 to 04/06/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

A review of the literature revealed that there is not yet a consensus on the definition of elementary (oo,1)-topos, but that the several proposed definitions of these objects are known to have an internal type theory resembling homotopy type theory (possibly with some restrictions on the type constructors). We therefore focussed on the special case of so-called Grothendieck (oo,1)-toposes and worked to identify the fragment of this language respected by geometric morphisms (transformations between these categories).

The majority of the theory of (oo,1)-toposes was developed by lifting results from the theory of 1-topos theory. A class of results which have not yet been lifted are those relating to **classifying toposes**. The reason for this is that there is an obstruction: the theory of classifying toposes relies on the fact that every Grothendieck 1-topos can be presented by a **site**, a fact which famously fails for (oo,1)-toposes, although a subclass of infinity-toposes admit such a presentation. To resolve this, we need a precise presentation of the data determining infinity-toposes; this led us to study the work of Anel, Biedermann, Finster and Joyal.

In the process of studying this obstruction we noticed that the (oo-connected, truncated) factorization system and its relation to “hypercompleteness” of these toposes play a crucial role in distinguishing “sheaf” oo-toposes from general ones, in the sense that any topos can be obtained from a sheaf topos by inverting oo-connected ones. To understand these better, we examined both localisations and

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

colocalisations of toposes.

The visiting researcher had the opportunity to meet with Gijs Heuts and Ieke Moerdijk, but due to conferences coinciding with their visit, there were limited opportunities for more prolonged interactions.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

oo-topos theory lies neatly at the interface of the involved researchers' interests, and we made a great deal of progress understanding the state of the art in the subject from our respective points of view. We intend to follow this work through to its natural conclusion, with an eventual publication attached. We both hope this collaboration will continue into the future and lead to further visits on both sides. In particular, a future visit to Utrecht by Morgan Rogers and a visit to LIPN (Paris) by Paige are natural follow-up steps.

This project contributes to the Types strand of the EuroProofNet action by providing external perspectives on the features and interactions that type systems reflecting existing and developing mathematical practices should have. In particular, the long term goal of this project is to understand the inference rules for constructing proofs in an (oo,1) version of geometric logic, which will be a fragment of Homotopy Typy Theory.