

## Short-Term Scientific Mission Grant - APPLICATION FORM<sup>1</sup> -

Action number: CA20111

Applicant name: Michele De Pascalis

### **Details of the STSM**

Title:  $\omega$ -categories, syntax and models

Start and end date: 14/01/2023 to 20/01/2023

### **Goals of the STSM**

Purpose and summary of the STSM.

*(max.200 word)*

$\omega$ -categories are an elusive subject in the current research surrounding category theory and type theory: although recurring patterns in the field suggest that such an abstraction should arise, it is still unclear how to develop the theoretical foundations for it.

Finster and Mimram [3] presented a type theory that was later stated ([1, unpublished]) to describe precisely  $\omega$ -categories as defined by Grothendieck and Maltsiniotis ([4, unpublished]). This type theory is still quite essential compared to the articulated type theory that Homotopy Type Theory is based on, that is, intensional dependent type theory à la Martin-Löf [5]. For instance it does not feature dependent sums or products, or exponential type constructors, therefore making it insufficiently expressive to reason about  $\omega$ -categories with richer structures. A particularly compelling perspective would be to be able to formulate a directed notion of path induction: as Escardó pointed out [2], there is a strong relation between path induction in Homotopy Type Theory and the Yoneda lemma.

The goal of this mission is therefore to gather first-hand information about the current state of research on syntaxes for  $\omega$ -categories, and to discuss a path towards the aforementioned extensions.

### **Working Plan**

Description of the work to be carried out by the applicant.

*(max.500 word)*

The subject of this work is mainly theoretical, so the research praxis is not particularly prone to methodological issues. Apart from presentation, discussion and general whiteboard activity, formal methods such as the use of proof assistants are expected to be employed. In particular, the type theory presented in [3] was later implemented in a couple of typecheckers<sup>23</sup>.

<sup>1</sup> This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

**Expected outputs and contribution to the Action MoU objectives and deliverables.**

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

*(max. 500 words)*

This work will be relevant to the research scope of Working Group 6 (Type Theory).

Eric Finster has been at the forefront of research regarding syntaxes for  $\omega$ -categories. By taking advantage of his expertise and exchanging ideas, I intend to gain a foothold in understanding constructive treatments of higher category theory, related type theories, with the intention of investigating ways to approach directed homotopy type theory. In particular, I seek to understand how the development of directed type theories can enable a synthetic and constructive formulation of the core items of category theory. Clearly if such a type theory is developed and implemented, it would encourage and facilitate a broader formalization of category theory.

<sup>2</sup> <https://github.com/ericfinster/catt>

<sup>3</sup> <https://github.com/smimram/catt>