Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Jacob Neumann

Details of the STSM

Title: Second-Order Generalised Algebraic Theories for Modal and Substructural Type Theories

Start and end date: 22/06/2024 to 06/07/2024

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The purpose of this STSM is to explore the variations and limitations of second-order generalised algebraic theories (SOGATs) as a framework for specifying type theories. The theory of SOGATs was introduced by Uemura [10] as a way of defining type theories which automates the bureaucracy of contexts and substitutions. The authors of [6] (including the proposed host for this STSM, Ambrus Kaposi) demonstrate that SOGATs can be used to define any structural type theory, by way of a theory of signatures for SOGATs. Moreover, techniques such as Synthetic Tait Computability [9] and internal sconing [3] are available for proving metatheoretic properties of these languages, like canonicity and normalisation.

However, further work is needed to understand how (or whether) substructural mechanics can be represented in the SOGAT framework. Some possibilities include converting substructural languages into structural languages by internalising the substitution calculus as explicit operations (as suggested by [1]), or instead utilising a substructural internal language (as in [4]). Our goal will be to explore these approaches, and gauge whether they're applicable to various substructural type theories. Some areas of interest include linear type theory, multi-modal type theory [5], and directed type theory. In general, we hope to better define the scope and limitations of SOGATs, and facilitate their deployment in wider areas of study.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

The visitor, Jacob Neumann, is a PhD student at the University of Nottingham (UK) studying directed

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type theory, specifically the application of SOGAT methods to directed type theory [7,8]; his forthcoming talk at the Working Group 6 meeting (titled "Towards Modal SOGATs") is intended as a precursor to this STSM. The host, Ambrus Kaposi, is an associate professor at Eötvös Loránd University (Hungary) and is one of the foremost experts on SOGATs and their use in the semantics of type theory [1,3,6]. The STSM will also overlap with a planned visit to Eötvös Loránd University by Thorsten Altenkirch, who is Neumann's PhD supervisor and a frequent collaborator with Kaposi, particularly on topics related to SOGATs. Several other researchers at Eötvös Loránd University have expert knowledge on this topic, such as Rafaël Bocquet [2,3] and Szumi Xie [6], and may collaborate on this STSM as well.

Our research during (and after) the STSM will focus on several key questions.

1. How can a SOGAT presentation be given for *particular* modal/substructural type theories of interest, such as polarised/directed type theory, multi-modal type theory, linear type theory, etc.?
2. What *general* patterns can be abstracted from these examples and existing instances (e.g. [1])? How can the theory of signatures and SOGAT-to-GAT translation of [6] be expanded to accommodate this general theory?
3. What substructural languages are not amenable to (concise and tractable) SOGAT representation? What properties do these theories have which prevent such a representation, and can an equivalent or near-equivalent theory be found which avoids this difficulty?
4. How can the methods discovered here be made readily applicable for researchers studying a wide variety of formal theories? Can SOGATs and the theory surrounding them be made understandable for students?

Throughout, our study will be informed by (and oriented towards) computer formalisation, with Agda being our common formal language. Accordingly, we will adopt a mix of informal, pseudo-formal, and formal styles to express our results.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

This STSM falls squarely within the mission of Working Group 6: SOGATs are a powerful and modular framework for reasoning about the syntax and semantics of type theories, and the aim of this STSM is to expand the range of languages to which SOGAT methods are applicable.

This STSM primarily addresses the EuroProofNet's Deliverable 4, in that it bolsters our mathematical tools for reasoning about important extensions to standard type theory. Representing type theories as second-order generalised algebraic theories seeks to strike a balance between more syntactic and more semantic approaches. SOGATs have a rich semantics in presheaf categories, and moreover the translation from SOGATs to GATs allows for any type theory given as a SOGAT to be studied using the semantic framework of categories with families. But SOGATs also function as a higher-order abstract syntax for specifying type theories, and have an attending theory of signatures. Much of the work in the theory of SOGATs has gone into understanding the mathematical interplay of syntax and semantics that they make possible. This STSM's focus on substructural constructs is likewise deeply tied to both

syntax and semantics: being 'substructural' is traditionally a syntactic notion—not admitting certain structural rules to the syntax—but their motivation is frequently quite semantic. We plan on developing further the mathematical theory understanding how such extensions operate.

This STSM also touches on several of the Action's objectives.

- **Research Coordination Objective 7**: as mentioned, SOGATs are a highly-modular framework, permitting the specification of type theories with any number of different possible configurations. This STSM seeks to expand this modularity to encompass substructural elements.
- **Capacity Building Objective 3**: the task of expanding the theory of SOGATs to include substructural languages is a considerable one, which will require extensive collaboration past the end of the STSM and potentially years into the future. Moreover, this project is fundamentally concerned with amassing more examples of languages and encompassing them into a common framework, meaning that researchers from various disciplines and communities have something to contribute to this project.
- **Capacity Building Objective 8**: we anticipate that this collaboration will yield results which we will seek to publish in appropriate computer science conferences (e.g. POPL, LICS, FSCD) and present at various conferences and workshops. Moreover, as indicated in question (4) above, part of the impetus for this STSM is towards making SOGATs more widely-accessible to the broader scientific community. We anticipate creating some written and video resources to this effect.

References

[1] Thorsten Altenkirch, Yorgo Chamoun, Ambrus Kaposi, and Michael Shulman. Internal parametricity, without an interval. Proc. ACM Program. Lang., 8(POPL):2340–2369, 2024.

[2] Rafaël Bocquet. External univalence for second-order generalized algebraic theories. CoRR, abs/2211.07487, 2022

[3] Rafaël Bocquet, Ambrus Kaposi, and Christian Sattler. For the metatheory of type theory, internal sconing is enough. In Marco Gaboardi and Femke van Raamsdonk, editors, 8th 689 International Conference on Formal Structures for Computation and Deduction, FSCD 2023, July 3-6, 2023, Rome, Italy, volume 260 of LIPIcs, pages 18:1–18:23. Schloss Dagstuhl - Leibniz Zentrum für Informatik, 2023.

[4] Daniel Gratzer. Normalization for multimodal type theory. In Christel Baier and Dana Fisman, editors, LICS ’22: 37th Annual ACM/IEEE Symposium on Logic in Computer Science, Haifa, Israel, August 2 - 5, 2022, pages 2:1–2:13. ACM, 2022.

[5] Daniel Gratzer, GA Kavvos, Andreas Nuyts, and Lars Birkedal. Multimodal dependent type theory. In Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science, pages 492–506, 2020.

[6] Ambrus Kaposi and Szumi Xie. Second-order generalised algebraic theories: signatures and first-order semantics. Submitted for publication, 2024. URL: https://akaposi.github.io/sogat.pdf

[7] Jacob Neumann. Deeply-polarized type theory as a generalized algebraic theory. Submitted for publication, 2024. URL: https://jacobneu.github.io/research/preprints/polarizedGAT.pdf

[8] Jacob Neumann. Presheaf semantics of polarized higher order abstract syntax. Second International Conference on Homotopy Type Theory, May 22 - 25, 2023, Pittsburgh, USA

[9] Jonathan Sterling. First Steps in Synthetic Tait Computability: The Objective Metatheory of Cubical Type Theory. PhD thesis, Carnegie Mellon University, USA, 2022.

[10] Taichi Uemura. Abstract and Concrete Type Theories. PhD thesis, University of Amsterdam, 2021.