Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Jasper Nalbach

Details of the STSM

Title: SMT solving and proofs for non-linear arithmetic

Start and end date: 28/08/2023 to 08/09/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

- *We deepened our understanding of the relation of the proof system with the coverings method.*
- *We discussed about the challenges for formally verifying the CAD and related methods. We also studied the BKR method for solving real algebra, which has been formally verified in a recent PhD thesis [2].*
- *We discussed various extensions of the high-level proof system to increase its efficiency for some cases, thus extending its scope for applicability. Some ideas are already worked out, some new ideas came up. In particular, we investigated how spurious factors in the iterated resultant can be omitted in the CalC method and the proof system, following a recent publication of James Davenport and Matthew England [1]. We briefly discussed the completeness of the proof system and Lazard's projection operator; however, as there is work on another complete CAD projection going on which might be simpler to integrate,we decided to wait for that work to be completed.*
- *As this was a joint visit with Lucas Michel, we also discussed mathematical ideas to deepen our understanding, and which may lead to further improvements to our work on proofs for real algebra.*

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

[1] Davenport, James H., and Matthew England. 'Iterated Resultants in CAD'. arXiv, 31 July 2023. http://arxiv.org/abs/2307.16750.

[2] Kosaian, Katherine. 'Formally Verifying Algorithms for Real Quantifier Elimination'. PhD Thesis, Carnegie Mellon University, 2023. http://reports-archive.adm.cs.cmu.edu/anon/2023/CMU-CS-23-130.pdf.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The main aim of this STSM was to catch up and synchronize the work done by Jasper Nalbach's work on the proof system, the plans of James Davenport on formal proofs, and the work on CAD by James Davenport and Matthew England, which was achieved. We will continue our collaboration on these topics. This STSM is in particular the start of the work on integrating the idea of exploiting iterated resultants into the proof system. This work will decrease the complexity of the computed proofs, and thus makes the formal verification of proofs feasible for more formulas.

Lucas Michel has a theoretical focus, while Jasper Nalbach works on integrating theoretical results into algorithms. During the stay, both profited from exchanging ideas on the CAD during the stay. Both plan to continue these discussions, also on applications and extensions of the proof system. This stay initiated this exchange, which may lead to contributions to the Action's goals in future.