Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Lucas MICHEL

Details of the STSM

Title: Proof calculus for Real Algebraic Geometry

Start and end date: 28/08/2023 to 08/09/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

This STSM gave me the opportunity to get a better understanding of the challenges, techniques, and unexplored areas for non-linear reasoning, production of proofs for non-linear arithmetics and related topics. Much of the discussions of these two weeks focused on tools for developing new results in these frameworks, namely the cylindrical algebraic decomposition (CAD) and related methods. Both theory and practice were discussed.

At Bath, we had numerous discussions concerning the current challenges concerning the CAD. This has led to new practical and theoretical ideas to extend the state-of-the art solution to the piano mover's problem, with recent techniques such as the cylindrical algebraic coverings [1]. We discussed about the challenges for formally verifying the CAD and related methods. We also studied the Ben-Or, Kozen and Reif's method for solving first-order real arithmetic, which has been formally verified in a recent PhD thesis from K.Kosaian [2].

At Coventry, we continued most of the discussions started with J.Davenport in Bath. We worked to understand the state-of-the-art algorithms to obtain the adjacency of CAD cells. We discussed the proof system [3] introduced by J.Nalbach, and we deepened our understanding of the relation of this proof system with the coverings method. New ideas, using geometric techniques, have emerged to potentially solve current challenges. I also had the opportunity to discuss with Tereso Del Rio, a PhD student from Coventry, about his current research on using machine learning techniques to speed-up algorithms in symbolic computations.

During these two weeks, I talked with J.Davenport and M.England about my ongoing research concerning the idea of a "best CAD adapted to a semi-algebraic set". Together with practical algorithms

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

to deduce the adjacency between CAD cells, this could lead to an effective way to simplify, in a canonical way, proof output in the context of non-linear arithmetic reasoning.

This scientific mission already led me to state and analyse new results concerning CAD, but also concerning the piano mover's problem. The results, whose proofs are currently being checked, will certainly lead to scientific publications. Moreover, this research stay will lead to further fruitful collaborations. Together with J. Nalbach, we plan to hold regular discussions to continue the ongoing research work started during this STSM.

[1] Erika Ábrahám, James H. Davenport, Matthew England, Gereon Kremer, "Deciding the consistency of non-linear real arithmetic constraints with a conflict driven search using cylindrical algebraic coverings", JSC, 2021, https://doi.org/10.1016/j.jlamp.2020.100633

[2] Kosaian, Katherine. 'Formally Verifying Algorithms for Real Quantifier Elimination'. PhD Thesis, Carnegie Mellon University, 2023. http://reports-archive.adm.cs.cmu.edu/anon/2023/CMU-CS-23-130.pdf.

[3] Jasper Nalbach, Erika Ábrahám, Philippe Specht, Christopher W. Brown, James H. Davenport, Matthew England, "Levelwise construction of a single cylindrical algebraic cell", preprint on arXiv, 2022, https://doi.org/10.48550/arXiv.2212.09309

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

Thanks to this STSM, L. Michel had the opportunity to meet world experts on the domain and he has been introduced to current challenges on non-linear reasoning. He plans to continue his research on minimum CAD.

This STSM was also a joint visit with J.Nalbach, who has a focus on integrating theoretical results into concrete algorithms implemented in SMT solvers. Both profited from exchanging ideas on CAD during the stay. They plan to continue these discussions in three main ways:

- applications and extensions of the J.Nalbach's proof system.
- applying recent techniques from the Satisfiability Checking and Symbolic Computation (SC²) communities to give better solutions to the piano mover's problem.
- Using geometric techniques to enhance the computer representation of semi-algebraic functions.

This stay initiated this exchange, which may lead to contributions to the Action's goals in the future.

The planned follow-up activities serve the same goal: developing techniques and tools for production of proofs from SMT solvers (and automated reasoners in general) when non-linear arithmetic reasoning is involved. These general goals contribute in particular to the EPN Objective 2 (Promote the output of detailed, checkable proofs from automated theorem provers), and D4 (Software for translating proof formats used by automated theorem provers to Dedukti).