Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: András Kovács

Details of the STSM

Title: Evaluation efficiency in cubical type theories

Start and end date: 09/10/2022 to 16/10/2022

Goals of the STSM

Motivation

Cubical type theories provide a highly expressive foundation for mechanized mathematics and verified programming. They support computational univalence, a powerful tool of generic reasoning about data structures and mathematical structures in general. However, existing implementations all suffer from performance issues that are often needed to be worked around. There are also definitions which are of mathematical interest but which cannot be efficiently computed in any existing system.

Goals

- First-principles investigation of efficiency of cubical evaluation, using small prototype implementations and adapting benchmarks by Ljungström and Mörtberg [1].

- If this yields significant optimizations, investigating their implementation in Cubical Agda, which is currently the most mature and popular implementation of a cubical type theory.

[1]: https://drops.dagstuhl.de/opus/volltexte/2022/15731/pdf/LIPIcs-CSL-2022-11.pdf

---

Working Plan

The activities performed during the STSM will depend on how much preliminary work I will be able to do before the visit. As of now, I plan to implement a prototype cubical type theory which includes several optimizations, before the STSM. However, it is hard to estimate exactly how much work it would take to get to the point where the mentioned benchmarks can be adapted. Hence, I describe two scenarios, depending on the readiness of the prototype.

1. *At the start of the STSM, there is a working prototype in which most benchmarks of interest can be implemented.* In this case, the tasks are:

- ○ Implementing benchmarks.

- ○ Implementing different design variations in the prototype and comparing them. Preliminarily, it seems that there are many potential optimizations that can be combined in a large number of ways, and we would like to isolate the essential optimizations if there are any.

- ○ Reviewing the Agda codebase, to figure out how optimizations might be added to it. I don't think that we can realistically make significant changes to the Agda codebase in the STSM week, but it might be still worth to do some groundwork.

2. *There is no working prototype at the start of the STSM.* In this case, the main task would be to try to make as much as possible progress in the implementation.

Expected outputs and contribution to the Action MoU objectives and deliverables.

1. Expected outputs

I expect to make significant progress on the problem of cubical evaluation efficiency. Preliminary analysis shows that existing implementations all use certain naive solutions which are likely to degrade performance, and it should be possible to make significant improvements. Also, the STSM participants have relevant competence in this area. My host, Anders Mörtberg, made the first practically usable cubical type theory implementation [2], and also coauthored several foundational papers in cubical type theory. I personally have experience in implementation and performance optimization of non-cubical type theories [3].

2. Contribution to action objectives

The contribution is aimed to improve the practical usability of cubical type theories. I assess that this is most relevant to the stated objective of **Working Group 6**, which is developing theories of type theories. While our contribution does not directly aim at this, note that cubical type theory is important as a metalanguage which can host many mathematical developments, including the WG6 objectives. Univalence and higher inductive types make cubical type theories some of the most expressive foundational theories available today. In particular, higher inductive and quotient inductive types are valuable tools in formalizing the general metatheory of type theories [4], and Cubical Agda is the only major system which supports them, but performance issues significantly limit their usefulness.

[2]: https://github.com/mortberg/cubicaltt

[3]: https://github.com/AndrasKovacs/smalltt

[4]: https://dl.acm.org/doi/10.1145/2837614.2837638