Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Maribel Fernandez

Details of the STSM

Title: Representation of Proofs via Hierarchical Higher-Order Port Graphs

Start and end date: 12/09/2023 to 19/09/2023

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

Our long-term goal is to develop a graph-based tool for the representation, analysis and management of proofs in various systems, using graph rewriting (a general specification model with formal semantics). Towards this goal, we designed graph-based representations of proofs in intuitionistic logic during the past year, and in this STSM our objective was to advance the work as described in the proposal. In particular, our specific objectives were:
- To specify proof management algorithms using hierarchical port graphs, first for intuitionistic logic and then for powerful logical frameworks (such as LF modulo, which is the basis of Dedukti).
- Plan the next steps towards the design of a domain-specific language for proof representation.

According to the proposed work plan, we designed hierarchical port graph rewriting rules to specify operations on intuitionistic logic proofs (e.g., proof normalisation), and defined strategies to control the rules to achieve expected results (e.g., to ensure the uniqueness of normal forms). This is the basis for the representation of more involved logics based on dependently-typed lambda-calculus, which are used in a number of proof assistants.
More precisely, the activities carried out during the visit involved:

1. Reviewing existing work on graph-based proof representations, in particular work done by Acclavio et al. on proof systems on graphs (Logical Methods in Computer Science Volume 18, Issue 4, 2022) and by Guerrieri et al. on representations of proof nets for linear logic (Logical Methods in Computer Science Volume 18, Issue 2, 2022)

2. Reviewing encodings of the lambda-calculus using hierarchical higher-order port graphs. By the

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Curry-Howard isomorphism, this gives direct representations of intuitionistics proofs.

3. Based on the above, we defined the general methodology to be used to design a domain-specific graph language for the representation of proofs. In particular, we identified the main components for a generic language for proof representation and manipulation, which is the first step towards the design of a domain-specific version of a graph-based modelling tool.

4. We created a plan for the rest of the work, which we plan to continue via regular online meetings and an in-person visit of Sandra Alves to London this Autumn.

Description of the STSM main achievements and planned follow-up activities

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

The visit achieved the planned goals and outcomes.

- The review of graph-based proof representations (see items 1 and 2 in the previous section) has provided the material for a journal article describing intuitionistic-proof representations using hierarchical higher-order port graphs, which we expect to be completed in 2024. We have already written a preliminary version, but more work is needed to test this approach and compare it with other related approaches (using for example bigraphs).
- Based on the work done in relation with item 3 above, we plan to write another article describing a general language for proof representation. So far, we have listed the main components:
    - for representation: syntax, types, deduction rules, simplification rules; and
    - for construction and management of proofs: configurations, proof construction steps, strategy.
- Plan for future work: first, we will test the methodology by developing more case studies (SLD-resolution systems, as well as standard natural deduction and sequent calculus systems). We will also check other generic tools for proof representation and management, such as Maude (based on rewriting logic: it is a general purpose programming language, which can be used to specify proof systems, although in a textual syntax rather than a graph-based one), K (based on matching logic), MLSOS (a metalanguage defined by Lakin and Pitts), amongst others.
- We plan to continue this work to develop the foundations for a graph-based proof management environment (in the style of PORGY but specifically tailored to the management of proofs) in a follow up visit of Sandra Alves to London in November 2023 to complete the specification of graph transformation rules implementing proof simplification. In addition, we are planning to involve undergraduate and postgraduate students from the University of Porto and King's College London in this collaborative work: we will propose topics for final-year BSc and MSc projects in Porto and London towards the above-stated goals, combining theoretical work with software implementation.