

Short-Term Scientific Mission Grant - APPLICATION FORM¹ -

Action number: CA20111

Applicant name: Thiago Felicissimo Cesar

Details of the STSM

Title: Compiling dependent pattern matching to elimination principles in Dedukti

Start and end date: 08 / 05 / 2022 to 21 / 05 / 2022

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

This STSM addresses specifically the issue of how Inductive Types are represented in Dedukti, and more particularly in the case of the translation from Agda to Dedukti. Most proof assistants feature a presentation of Inductive Types in which terms are defined by dependent pattern matching instead of by using the elimination principles directly. In the present, Dedukti translators, such as Agda2Dedukti, translate these definitions directly as rewriting rules. However Dedukti is not able to check that such definitions are terminating or complete. Moreover, this also poses an interoperability problem: a pattern-matching definition accepted by Agda won't necessarily be accepted by Coq, because their termination checkers have different conditions. To solve this problem, we propose to compile in Dedukti the definitions by pattern-matching into definitions using only elimination principles, which would give complete and terminating definitions by construction. Fortunately, there is already literature on this problem, and in particular Jesper Cockx (who will be the host of this mission) has proved this can be done in Agda. The goal of this STSM is to do a first implementation of such a compilation in Dedukti (using the work in "Pattern Matching Without K"), with the goal of addressing the aforementioned issues.

Working Plan

Description of the work to be carried out by the applicant.

¹ This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via-e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

(max.500 word)

Implementing the discussed transformation is expected to demand a high workload, so in this mission we will focus on doing a first prototype by building it over the Agda2Dedukti translator, as a first step.

The compilation of pattern-matching can be described by the following four stages.

1. Definitions by user-written pattern-matching clauses
2. Terms built using case trees
3. Compiled terms using case analysis, well-founded induction, no-confusion principle and acyclicity principle
4. Compiled terms using only elimination principles

Today, Agda2Dedukti produces files which are the direct translation of the user-written clauses (1). By building our tool on top of Agda2Dedukti, which is integrated with Agda, we expect to be able to reuse some of the functionalities that are already implemented in Agda. In particular, Agda implements internally the elaboration phase, which performs the translation from (1) to (2).

The heart of the implementation would then be the transformation from (2) to (3). This consists of compiling the case trees --- which are not necessarily well-founded, and thus also need to pass a termination checker --- into terms which use specialized versions of the elimination principles. More precisely, the definition of these terms use principles such as well-founded induction and no-confusion, which can be derived by the elimination principle. At this stage, if we assume the derivability of these principles (which has already been shown by pen and paper proofs), then a definition checked correct by Dedukti will be terminating and complete by construction.

The last step, which consists of the transformation from (3) to (4), aims at eliminating the trust on the derivability of these principles by automatically building their proofs.

As we intend to build a first prototype, we do not plan on implementing all these steps immediately, in particular the transformation from (3) to (4). The transformation from (1) to (2) should also be made outside Agda in the future, in order for the tool to be usable across different Dedukti translators.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

This mission will allow for the development of a first prototype to compile dependent pattern matching definitions to ones using only the elimination principles in Dedukti. This development will continue after the mission, in order to implement all the functionalities we envision. In particular, even thought as a first step we will focus on definitions coming only from Agda, in the future we expect this tool to be translator-independent, which would allow it to be used with all the other Dedukti translators, such as VoDk (from Coq to Dedukti).

The main benefit from this work is to provide a way such that these definitions can be fully verified by Dedukti. Indeed, as already mentioned, Dedukti cannot check if a definition by pattern matching is terminating, and in the case of translated Agda files it cannot even check that a given definition is complete (that is, covers all constructors). Moreover, this transformation will also be useful for checking that a definition by pattern matching can indeed be translated to the intended core type theory, without additional principles such as Uniqueness of Identity Proofs. Finally, once a definition is compiled to elimination principles it can be accepted by any proof assistant featuring inductive types, even if its termination checker is not capable of verifying that the original definition by pattern-matching is terminating. Therefore, this work will help to share proofs across proof assistants based on type theory, and in particular also contribute to the deliverable "Release of software for translating proofs coming from important proof systems based on type theory like Isabelle, Agda, PVS, Lean or Minlog, to Dedukti and back".

Regarding capacity building objectives, as I am part of the Dedukti development team and Jesper is part of the Agda developpement team, this work will contribute to further strengthen the links between

those communities, and help to “Bring together members of the different communities working on proofs in Europe.”. Moreover, this will be the perfect opportunity for Jesper to share knowledge about Agda with me, and for me to share knowledge about Dedukti with him, thus contributing to “Transfer knowledge in terms of expertise, scientific tools and human resources across the different disciplines and between academia and industry.”. Finally, as I am a young 1st year PhD student, this mission will also contribute to “Actively support young researchers, the under-represented gender, and teams from regions with less capacity.”.