Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number:

Applicant name: Elena Di Lavore

Details of the STSM

Title: Bidimensional Markov categories

Start and end date: 11/03/2024 to 24/03/2024

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

This STSM aims to find syntax and categorical semantics for deriving bounds on the likelihood of success or hazard on the behaviour of probabilistic programs or systems. We plan to develop a syntax for a bicategorical analogue of Markov categories, containing both a formulation of synthetic probability (Markov categories) and proof-relevant bound construction (reflected on the 2-cells).

Markov categories [5,15,16] constitute a synthetic categorical formulation of probability theory. A recent extension by the host and visitor of this STSM, Partial Markov Categories [3], provides a synthetic theory of Bayes updates and a graphical syntax that can be translated to a simple type theory in terms of do-notation [17]. Still, these theories miss the ability to derive probabilistic bounds. In the same way that cartesian bicategories of relations [2] allow us to bound the possible outputs of a non-deterministic program, the bidimensional syntax developed from this STSM would allow us to express statements like 'if the pseudorandom generator used is indistinguishable from a true random generator with a negligible bound, the probability of a successful attack to a cryptographic protocol is less than a given bound'.

References

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[2] Carboni, Aurelio, and Robert FC Walters. "Cartesian bicategories I." *Journal of pure and applied algebra* 49.1-2 (1987): 11-32.

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

[3] Di Lavore, Elena and Mario Román, "Evidential Decision Theory via Partial Markov Categories," *2023 38th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*, Boston, MA, USA, 2023, pp. 1-14, doi: 10.1109/LICS56636.2023.10175776.

[4] Earnshaw, Matt, James Hefford, and Mario Román. "The Produoidal Algebra of Process Decomposition." *To be presented at Computer Science Logic* 2024.

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[6] Goy, Alexandre, and Daniela Petrişan. "Combining probabilistic and non-deterministic choice via weak distributive laws." *Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science*. 2020.

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[10] Di Lavore, Elena, Jules Hedges, and Paweł Sobociński. "Compositional Modelling of Network Games." *29th EACSL Annual Conference on Computer Science Logic (CSL 2021)*. Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.

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[12] Broadbent, Anne, and Martti Karvonen. "Categorical composable cryptography: extended version." *Logical Methods in Computer Science* 19 (2023).

[13] Di Lavore, Elena, Giovanni de Felice, and Mario Román. "Monoidal streams for dataflow programming." *Proceedings of the 37th Annual ACM/IEEE Symposium on Logic in Computer Science*. 2022.

[14] Day, Brian, Elango Panchadcharam, and Ross Street. "Lax braidings and the lax centre." *Contemporary Mathematics* 441 (2007): 1.

[15] Perrone, Paolo. "Markov categories and entropy." *IEEE Transactions on Information Theory* (2023).

[16] Fritz, Tobias, and Wendong Liang. "Free gs-monoidal categories and free Markov categories." *Applied Categorical Structures* 31.2 (2023): 21.

[17] Hughes, John. "Generalising monads to arrows." *Science of computer programming* 37.1-3 (2000): 67-111.

[18] Paquet, Hugo, and Philip Saville. "Effectful semantics in 2-dimensional categories: premonoidal and Freyd bicategories." *arXiv preprint arXiv:2312.14964* (2023).

[19] Fiore, Marcelo, and Philip Saville. "A type theory for cartesian closed bicategories." *2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*. IEEE, 2019.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

Practical aspects: The visitor and the host will work together, building on previous joint work [3]. An in-person visit would provide the time to collaborate on the whiteboard and tackle the key difficulties of the problem. Once these are settled, it will become easier to coordinate towards a complete text and an eventual paper submission.

Visitor and host bring different expertise to the proposal while sharing a common ground. The visitor (Elena Di Lavore) has already published on compositional approaches to program dataflow [9] and economic game theory [10]. The host (Mario Román) has published on string diagrammatic approaches to programming effects [11] and causal analysis of monoidal processes [4]. Both have already collaborated on a first extension of Markov categories to explicit Bayesian updates [3] and an application to dataflow programming [13]; in both cases, the results were published at Logic in Computer Science (LiCS). The plan for this STSM is to expand this project on Markov categories.

Technical aspects: We will propose and investigate a definition of Markov bicategory, where the two-dimensional structure allows us to reason about bounds. We will pursue a graphical calculus and a corresponding type theory for Markov bicategories. In particular, we will investigate the role of conditionals in this definition: a crucial aspect of Markov categories is how they capture causal dependence; we conjecture a categorical semantics of this dependence can be given in terms of a natural produoidal structure [4] on top of Markov bicategories, in a similar fashion to Shapiro and Spivak's study of dependence with duoidal structures [7].

We plan to capture examples of probabilistic reasoning in computer science. For instance, we will look at previous work by Broadbent and Karvonen [12], which provides a synthetic approach to cryptography with monoidal categories. If time permits, we will try to prove safety bounds on some simple cryptographic protocols.

Multiple topics on categorical semantics may be developed further due to this STSM. The semantic universes for our goal syntax would likely combine probability and non-determinism [1,6,8], and rely on a refined definition of conditional [5,3]. The causal structure of Markov categories may be synthetically captured by the notion of produoidal category [4,7,14]. Multiple variants of monoidal bicategories have been described, some recently (premonoidal, (adjoint-)cartesian, closed, or Freyd bicategories [2,18,19]); we expect to develop bidimensional Markov categories.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

We expect this research to fall within Deliverable D4 ("Definition of a mathematical framework for modular reasoning about type theories and their extensions"). A result of a better syntax for Markov bicategories will fall within Research Coordination Objective 3 ("Make techniques for program verification more effective and more accessible to all stakeholders").

Additionally, we would make progress towards the following Capacity Building Objectives: Objective 1 ("Bring together members of the different communities working on proofs in Europe") and Objective 3 ("Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the action"), by contributing to a lasting collaboration between postdocs at Pisa and Oxford; Objective 4 ("Ease access to formal verification techniques in education and other areas of science"), by proposing an intuitive syntax for probabilistic reasoning; and Objective 5 ("Actively support young researchers, the under-represented gender, and teams from regions with less capacity").

We aim to publish this work in a relevant theoretical computer science conference (e.g. ETAPS, LiCS, POPL) or journal (e.g. LMCS, MSCS). The development of Markov categories is additionally of interest to the category theory, compositional cryptography, and the probabilistic programming communities.