Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111

Applicant name: Stefania Damato

Details of the STSM

Title: Coinduction in Cubical Agda

Start and end date: 19/06/2024 to 28/06/2024

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

Cubical type theory and Cubical Agda have proven very useful for giving computational meaning to homotopy type theory and univalent foundations. This STSM focusses on the usefulness of Cubical Agda in the truncated (Set-level) setting in relation to coinduction, and our goal is to better support coinductive reasoning in Cubical Agda.

In vanilla Agda, making use of and proving statements about inductive types is facilitated by way of pattern matching and structural recursion. On the other hand, while we can define coinductive types, we cannot prove statements about them. Coinduction, like function extensionality, is not supported in intensional type theories like vanilla Agda. Cubical Agda partially solves this problem by replacing Martin-Löf's identity type with the path type. This allows us to prove coinductive results and seems to restore the symmetry with inductive types, such that coinductive proofs can be derived via copattern matching and guarded corecursion.

However, several issues remain. The `with` clause is still not properly supported, and the termination checker still complains in some cases even when a recursive call is in principle guarded.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

Our work will involve looking at different ways we can address the existing problems around the use of coinduction in Cubical Agda. We will look at workarounds involving the existing technology, as well as the theoretical framework that would be necessary for an alternative approach altogether. More specifically, we aim to:

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

- Write a suite of coinductive examples. I have already written a few examples showcasing the existing problems in Cubical Agda. We would like to expand these to a library of examples, from which we can extract patterns to guide us with the rest of our goals.
- Examine what theoretical results would be needed to modify the termination checker, such that it accepts transitivity proofs involving only recursive calls and primitives (like `hcomp`), in a way that keeps the system sound.
- Look into an alternative definition of equality on coinductive types. Such a definition would be (propositionally) equivalent to the Path type but would incorporate primitives that do not affect guardedness, like transitivity of equality proofs. We already have a candidate for this, but its properties still need to be checked.
- Work towards reducing the use of coinduction in Cubical Agda to a core principle, theory, or combinator. For inductive types, we know that the use of pattern matching is equivalent to using eliminators, and the eliminator itself can be derived from the non-dependent eliminator and its uniqueness. Does an equivalent result exist for coinductive types? In Cubical Agda currently, we can prove certain statements using the coeliminator which we cannot otherwise do via copattern matching. We'd like to explore if this is a theoretical restriction, or a result of the current termination checker.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

This STSM will contribute to the objectives of WG6 (Type Theory). Improving the formalisation of coinductive proofs is necessary if we want to be able to reason about infinite structures, such as processes that run indefinitely. Coinduction is a natural way of approaching this, especially in a proof assistant such as Cubical Agda. We aim to write up our findings and publish them in a relevant theoretical computer science conference or journal.

Our work contributes towards the following Research Coordination Objectives: writing a library of examples, fixing current obstacles that arise when working with coinductive types, and making sure these are better supported, falls under Objective 3 (Make techniques for program verification more effective and more accessible to all stakeholders); work towards reducing the use of coinduction to a core principle falls under Objective 7 (Develop a modular theory of type theories).