Short-Term Scientific Mission Grant
- APPLICATION FORM[1] -

Action number: CA20111 – European Research Network on Formal Proofs

Applicant name: Nathanael Arkor

Details of the STSM

Title: Towards 2-dimensional second-order algebraic theories

Start and end date: 10/03/2024 to 16/03/2024

Goals of the STSM

Purpose and summary of the STSM.

(max.200 word)

The purpose of this project is to develop a category-theoretic framework combining second-order algebraic theories and rewriting theory. Traditional algebraic theories, as studied in universal algebra, describe structures equipped with n-ary operations (which do not bind variables). Second-order algebraic theories, introduced by Fiore and Mahmoud [1] as a step in the programme of Algebraic Type Theory [2, 3, 4, 5, 6], generalise algebraic theories with variable-binding operations. Second-order algebraic theories capture simple examples of type theories. However, the formalism of second-order algebraic theories is only able to describe equalities between terms, not rewrites. In recent years, there has been significant interest in using higher categories to model program transformations (e.g. beta reductions and eta expansions) [7, 8, 9]. A category has objects and morphisms, which respectively model the types and terms-in-context of a type theory. A 2-category extends the structure of a category with 2-cells, which model rewrites between terms. 2-dimensional approaches thus facilitate the description of the computational aspects of a type theory.

Our objective is use 2-category theory to develop 2-dimensional generalisations of second-order algebraic theories and establish their theory. Our eventual goal is to provide a modular framework for describing type theories with variable-binding operations and rewriting rules, in the style of categorical algebra.

[1] Fiore, Marcelo, and Ola Mahmoud. "Second-order algebraic theories." Mathematical Foundations of Computer Science 2010: 35th International Symposium, MFCS 2010, Brno, Czech Republic, August 23-27, 2010. Proceedings 35. Springer Berlin Heidelberg, 2010.

[2] Fiore, Marcelo, and Chung-Kil Hur. "Second-order equational logic." Computer Science Logic: 24th International Workshop, CSL 2010, 19th Annual Conference of the EACSL, Brno, Czech Republic,

[1] This form is part of the application for a grant to visit a host organisation located in a different country than the country of affiliation. It is submitted to the COST Action MC via e-COST. The Grant Awarding Coordinator coordinates the evaluation on behalf of the Action MC and informs the Grant Holder of the result of the evaluation for issuing the Grant Letter.

August 23-27, 2010. Proceedings 24. Springer Berlin Heidelberg, 2010.

[3] Fiore, Marcelo, Gordon Plotkin, and Daniele Turi. "Abstract syntax and variable binding." Proceedings. 14th Symposium on Logic in Computer Science (Cat. No. PR00158). IEEE, 1999.

[4] Fiore, Marcelo. "Second-order and dependently-sorted abstract syntax." 2008 23rd Annual IEEE Symposium on Logic in Computer Science. IEEE, 2008.

[5] Arkor, Nathanael, and Marcelo Fiore. "Algebraic models of simple type theories: A polynomial approach." Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science. 2020.

[6] Arkor, Nathanael, and Dylan McDermott. "Abstract Clones for Abstract Syntax." 6th International Conference on Formal Structures for Computation and Deduction (FSCD 2021). Schloss Dagstuhl-Leibniz-Zentrum für Informatik, 2021.

[7] Licata, Daniel R., and Robert Harper. "2-dimensional directed type theory." Electronic Notes in Theoretical Computer Science 276 (2011): 263-289.

[8] Fiore, Marcelo, and Philip Saville. "A type theory for cartesian closed bicategories." 2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS). IEEE, 2019.

[9] Ahrens, Benedikt, Paige Randall North, and Niels van der Weide. "Bicategorical type theory: semantics and syntax." Mathematical Structures in Computer Science (2023): 1-45.

Working Plan

Description of the work to be carried out by the applicant.

(max.500 word)

Practical aspects

The visitor (Nathanael Arkor) and host (Dylan McDermott) will work together with Philip Saville (also applying for an STSM). This collaboration brings together expertise in each of the two themes of the proposal: Arkor and McDermott have previously collaborated on categorical algebra and second-order algebraic theories (cf. [6] above); and Saville is an expert on 2-dimensional categorical semantics (cf. [8] above).

Our work is at currently an early stage. Our plan is to use the time provided by the STSM to develop the foundational theory and to understand the fundamental definitions and theorems necessary for developing the theory. Being able to collaborate at a whiteboard in the same room for an extended period will be invaluable. Once we have developed the key ideas during this time, we will be able to effectively divide the remaining work and to each focus on different aspects of the project after the STSM has concluded.

Technical aspects

Our aim is to use 2-category theory to develop 2-dimensional generalisations of second-order algebraic theories and their presentations. We will begin by giving a definition of 2-dimensional second-order presentation, which describes the specification of a type theory with binding operators and rewriting rules. Next, we will show this definition is correct by proving theorems paralleling those that hold in the 1-dimensional setting. For example, it is known that the correspondence between (1) equational presentations, (2) algebraic theories, and (3) finitary monads in classical 1-dimensional universal algebra extends to the second-order setting (i.e. generalising algebraic operations to variable-binding operations). From a type-theoretic perspective, this says that we may express type theories by their (1) syntactic presentations, (2) in a presentation-free manner, (3) by certain monads on the category of algebraic theories. We will establish analogues of these results in the presence of rewriting.

Finally, if time permits, we will examine further generalisations using enriched category theory. 2-dimensional categories may be viewed as categories enriched in the category of small categories, but there are other bases of enrichment that are likely to prove useful for capturing the behaviour of type theories, e.g. suitable categories of topological spaces or metric spaces, which would provide a semantic model for type theories with a notion of distance or cost between terms, rather than rewrites. The 2-dimensional setting will be useful also for this direction, as 2-category theory forces the use of abstract and general arguments, which are frequently amenable to generalisation to other settings.

Expected outputs and contribution to the Action MoU objectives and deliverables.

Main expected results and their contribution to the progress towards the Action objectives (either research coordination and/or capacity building objectives) and deliverables.

(max.500 words)

We anticipate that our work will fall squarely within Deliverable D4 ("Definition of a mathematical framework for modular reasoning about type theories and their extensions") and Research Coordination Objective 7 ("Develop a modular theory of type theories").

The work in this STSM will be the starting point for not only this project, but also for an ongoing collaboration focused on developing category-theoretic techniques for the description of type theories and programming languages. This STSM will therefore also contribute to achieving the following Capacity Building Objectives:

1. Bring together members of the different communities working on proofs in Europe.

3. Create an excellent and inclusive network of researchers in Europe with lasting collaboration beyond the lifetime of the Action.

5. Actively support young researchers, the under-represented gender, and teams from regions with less capacity.

7. Prepare competitive EU researchers for a fruitful career in an international environment through intensive use of Short Term Scientific Missions (STSM) and joint educational programs with industry.

Finally, our aim is to publish the work resulting from this collaboration in a relevant computer science conference (e.g. ETAPS, LICS, POPL). The abstract category-theoretic developments may also be of significance in their own right (independent to the primary, type-theoretic motivation), in which case we aim to publish the developments in a mathematics journal. Together, this contributes to Capacity Building Objective 8 ("Disseminate the results of the Action activities to the scientific community, the industry, the certification bodies, the European institutions and to the general public").