STSM application for Initial Semantics for Polymorphic Type Systems

Title:

Initial semantics for polymorphic type systems

Goals:

During this STSM, we aim to work towards the goals of WG6.
The goal of this STSM is to develop initial semantics for polymorphic type systems. This means:

- Develop a notion of signature for polymorphic type systems
- Associate to any signature a category of models
- Identify sufficient conditions for signatures to admit an initial model
- Study the expressivity of the recursion principle stemming from initiality; in particular, what are the translations of languages that can be defined by recursion?

Working plan:

There are many works on initial semantics for untyped and simply-typed programming languages (e.g., PCF), but only a few extended abstracts without proofs for polymorphic type systems. Yet, polymorphic type systems present an important stepping stone from simply-typed to dependently-typed languages, which are the ultimate topic of WG6. The difficulty of polymorphic type systems, compared to simply-typed ones, is the varying type-level context, indicating the possibility of binding type-variables by, for instance, a type-level universal quantification (think as the type of the polymorphic identity function as forall X, X -> X).

In this STSM, we will follow an initial sketch developed by Benedikt Ahrens and Thomas Lamiaux during Lamiaux's internship with Ahrens in 2022/23. Specifically, we aim to study the transfer of typed signatures and their models across a translation of types, in the style of a Grothendieck fibration. We anticipate that through this "pullback" construction, we can handle the varying type contexts which make polymorphism difficult to capture mathematically.

Remark: while this is officially an STSM between Ahrens and Lafont, Lamiaux is going to participate in the work.

Expected output:

- A paper on initial semantics for polymorphic type systems (Deliverable: Definition of a mathematical framework for modular reasoning about type theories and their extensions.)
- Possibly a computer formalization (which would go some way towards a prototype implementation, Deliverable Prototype implementation of the mathematical framework)