TRAINING ENIGMAs, CoPs, AND OTHER THINKING CREATURES

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Leibniz's/Hilbert's/Russell's Dream: Let Us Calculate!

Solve all (math, physics, law, economics, society, ...) problems by reduction to logic/computation

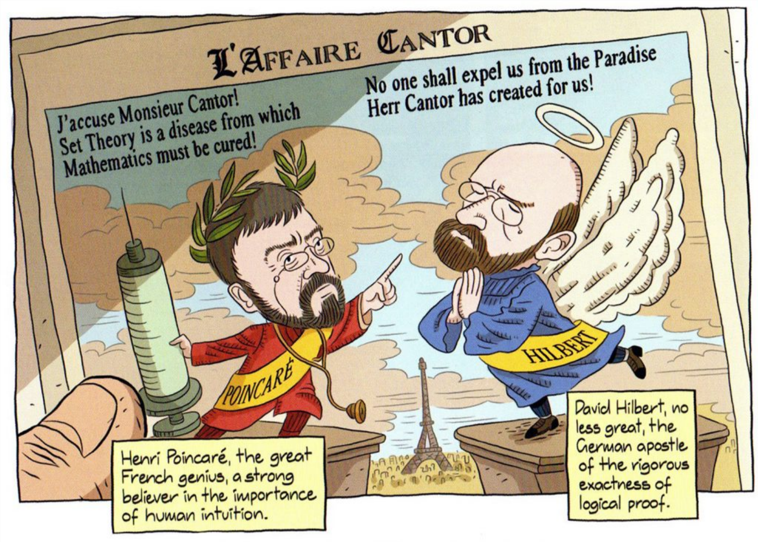
[Adapted from: *Logicomix: An Epic Search for Truth* by A. Doxiadis]

How Do We Automate Math, Science, Programming?

- What is mathematical and scientific thinking?
- Pattern-matching, analogy, induction from examples
- Deductive reasoning
- Complicated feedback loops between induction and deduction
- Using a lot of previous knowledge - both for induction and deduction

- We need to develop such methods on computers
- Are there any large corpora suitable for nontrivial deduction?
- Yes! Large libraries of formal proofs and theories
- So let's develop strong AI on them!

Intuition vs Formal Reasoning – Poincaré vs Hilbert



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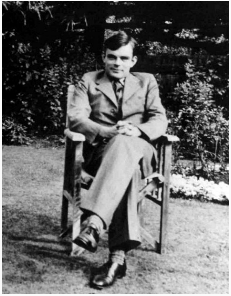
Induction/Learning vs Reasoning – Henri Poincaré

- Science and Method: Ideas about the interplay between correct deduction and induction/intuition
- *“And in demonstration itself logic is not all. The true mathematical reasoning is a real induction [...]”*
- I believe he was right: strong general reasoning engines have to combine deduction and induction (learning patterns from data, making conjectures, etc.)

Learning vs Reasoning – Alan Turing 1950 – AI



- 1950: *Computing machinery and intelligence* – AI, Turing test
- *"We may hope that machines will eventually compete with men in all purely intellectual fields."* (regardless of his 1936 undecidability result!)
- last section on Learning Machines:
- *"But which are the best ones [fields] to start [learning on] with?"*
- *"... Even this is a difficult decision. Many people think that a very abstract activity, like the playing of chess, would be best."*
- Why not try with math? It is much more (universally?) expressive ...

History and Motivation for AI/ML/TP

- Intuition vs Formal Reasoning – Poincaré vs Hilbert, Science & Method
- Turing's 1950 paper: Learning Machines, learn Chess?, undecidability??
- Lenat, Langley, etc: manually-written heuristics, learn Kepler laws,...
- Denzinger, Schulz, Goller, Fuchs – late 90's, ATP-focused:
- Learning from Previous Proof Experience
- My MSc (1998): Try ILP to learn explainable rules/heuristics from Mizar
- Since: Use large formal math (Big Proof) corpora: Mizar, Isabelle, HOL
- ... to combine/develop symbolic/statistical deductive/inductive ML/TP/AI
- ... hammer-style methods, feedback loops, internal guidance, ...
- More details – AGI'18 keynote: `https://bit.ly/3qifhg4`
- AI vs DL: Ben Goertzel's Prague talk: `https://youtu.be/Zt2HSTuGBn8`
- Big AI visions: automate/verify math, science, law, (Leibniz, McCarthy, ..)
- Practical impact: boost today's large ITP verification projects

Why Do This Today?

1. Practically Useful for Verification of Complex HW/SW and Math
    - Formal Proof of the Kepler Conjecture (2014 – Hales – 20k lemmas)
    - Formal Proof of the Feit-Thompson Theorem (2 books, 2012 – Gonthier)
    - Verification of several math textbooks and CS algorithms
    - Verification of compilers (CompCert)
    - Verification of OS microkernels (seL4), HW chips (Intel), transport, finance,
    - Verification of cryptographic protocols (Amazon), etc.
2. Blue Sky AI Visions:
    - Get strong AI by learning/reasoning over large KBs of human thought?
    - Big formal theories: good semantic approximation of such thinking KBs?
    - Deep non-contradictory semantics – better than scanning books?
    - Gradually try learning math/science
    - automate/verify them, include law, etc. (Leibniz, McCarthy, ..)
        - What are the components (inductive/deductive thinking)?
        - How to combine them together?

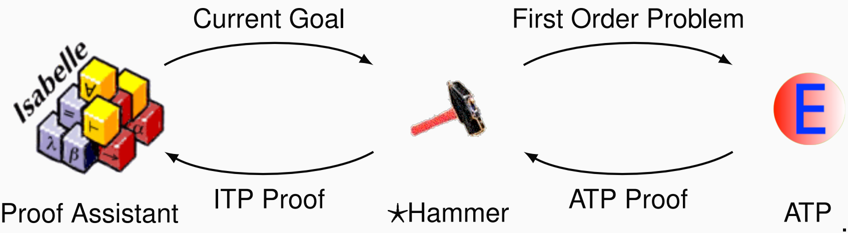
Using Learning to Guide Theorem Proving

- **high-level**: pre-select lemmas from a large library, give them to ATPs
- **high-level**: pre-select a good ATP strategy/portfolio for a problem
- **high-level**: pre-select good *hints* for a problem, use them to guide ATPs
- **low-level**: guide every inference step of ATPs (tableau, superposition)
- **low-level**: guide every kernel step of LCF-style ITPs
- **mid-level**: guide application of tactics in ITPs
- **mid-level**: invent suitable ATP strategies for classes of problems
- **mid-level**: invent suitable conjectures for a problem
- **mid-level**: invent suitable concepts/models for problems/theories
- **proof sketches**: explore stronger/related theories to get proof ideas
- **theory exploration**: develop interesting theories by conjecturing/proving
- **feedback loops**: (dis)prove, learn from it, (dis)prove more, learn more, ...
- **autoformalization**: (semi-)automate translation from LaTeX to formal
- ...

AI/TP Examples and Demos

- ENIGMA/hammer proofs of Pythagoras : `https://bit.ly/2MVPAn7` (more at `http://grid01.ciirc.cvut.cz/~mptp/enigma-ex.pdf`) and simplified Carmichael `https://bit.ly/3oGBdRz`,
- 3-phase ENIGMA: `https://bit.ly/3C0Lwa8`, `https://bit.ly/3BWqR6K`
- Long trig proof from 1k axioms: `https://bit.ly/2YZ0OgX`
- Extreme Deepire/AVATAR proof of $\epsilon_0 = \omega^{\omega^{\omega^{\cdot^{\cdot^{\cdot}}}}}$ `https://bit.ly/3Ne4WNX`
- Hammering demo: `http://grid01.ciirc.cvut.cz/~mptp/out4.ogv`
- TacticToe on HOL4: `http://grid01.ciirc.cvut.cz/~mptp/tactictoe_demo.ogv`
- Tactician for Coq: `https://blaauwbroek.eu/papers/cicm2020/demo.mp4`, `https://coq-tactician.github.io/demo.html`
- Inf2formal over HOL Light: `http://grid01.ciirc.cvut.cz/~mptp/demo.ogv`
- QSynt: AI rediscovers the Fermat primality test: `https://www.youtube.com/watch?v=24oejR9wsXs`

Today's AI-ATP systems ($\star$-Hammers)



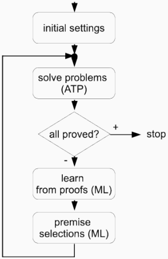
How much can it do?

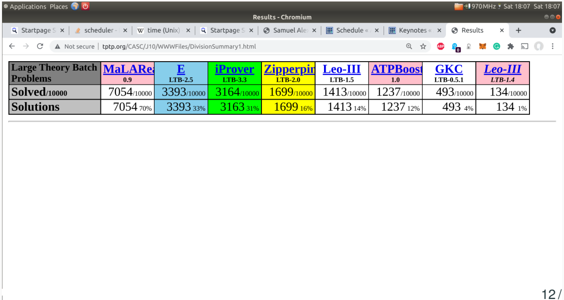
- Mizar / MML – MizAR
- Isabelle (Auth, Jinja) – Sledgehammer
- Flyspeck (including core HOL Light and Multivariate) – HOL(y)Hammer
- HOL4 (Gauthier and Kaliszyk)
- CoqHammer (Czajka and Kaliszyk) - about 40% on Coq standard library

$\approx$ 40-45% success by 2016, 60% on Mizar as of 2021

High-level feedback loops – MALARea, ATPBoost

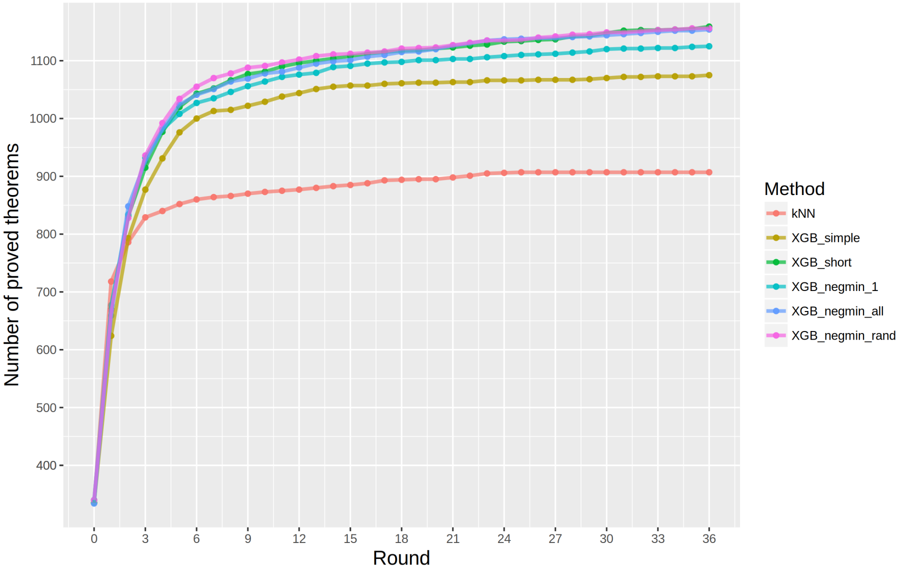
- Machine Learner for Autom. Reasoning (2006) – infinite hammering
- feedback loop interleaving ATP with learning premise selection
- both syntactic and semantic features for characterizing formulas:
- evolving set of finite (counter)models in which formulas evaluated
- winning AI/ATP benchmarks (MPTPChallenge, CASC 08/12/13/18/20)
- ATPBoost (Piotrowski) - recent incarnation focusing on multiple proofs

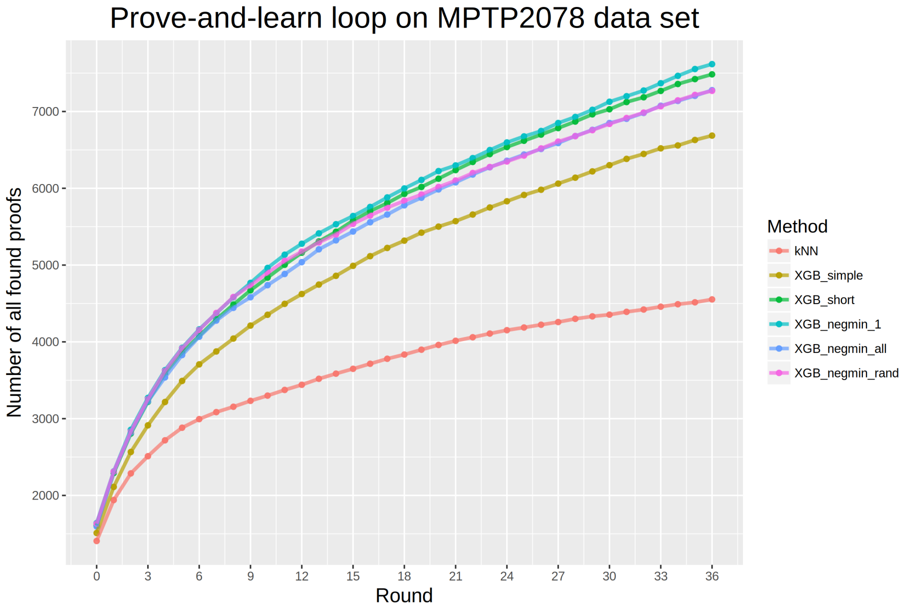


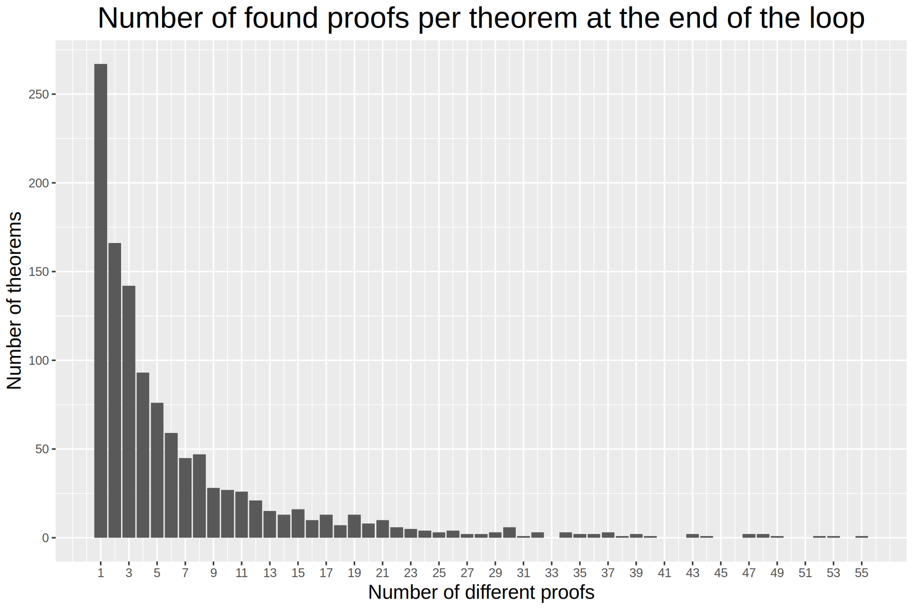


Large Theory Batch Problems	MaLARea 0.9	E LTB-2.5	iProver LTB-3.3	Zipperpin LTB-2.0	Leo-III LTB-1.5	ATPBoost 1.0	GKC LTB-0.5.1	*Leo-III LTB-1.4*
Solved/10000	7054/10000	3393/10000	3164/10000	1699/10000	1413/10000	1237/10000	493/10000	134/10000
Solutions	7054 70%	3393 33%	3163 31%	1699 16%	1413 14%	1237 12%	493 4%	134 1%

Prove-and-learn loop on MPTP2078 data set




Prove-and-learn loop on MPTP2078 data set
Number of all found proofs
7000
6000
5000
4000
3000
2000
0
3
6
9
12
15
18
21
24
27
30
33
36
Round
Method
kNN
XGB_simple
XGB_short
XGB_negmin_1
XGB_negmin_all
XGB_negmin_rand


Number of found proofs per theorem at the end of the loop
Number of theorems
Number of different proofs
0
50
100
150
200
250
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
55

FACE_OF_POLYHEDRON_POLYHEDRON

```
let FACE_OF_POLYHEDRON_POLYHEDRON = prove
 (`!s:real^N->bool c. polyhedron s /\ c face_of s ==> polyhedron c`,
  REPEAT STRIP_TAC THEN FIRST_ASSUM
   (MP_TAC o GEN_REWRITE_RULE I [POLYHEDRON_INTER_AFFINE_MINIMAL]) THEN
  REWRITE_TAC[RIGHT_IMP_EXISTS_THM; SKOLEM_THM] THEN
  SIMP_TAC[LEFT_IMP_EXISTS_THM; RIGHT_AND_EXISTS_THM; LEFT_AND_EXISTS_THM] THEN
  MAP_EVERY X_GEN_TAC
   [`f:(real^N->bool)->bool`; `a:(real^N->bool)->real^N`;
    `b:(real^N->bool)->real`] THEN
  STRIP_TAC THEN
  MP_TAC(ISPECL [`s:real^N->bool`; `f:(real^N->bool)->bool`;
                `a:(real^N->bool)->real^N`; `b:(real^N->bool)->real`]
        FACE_OF_POLYHEDRON_EXPLICIT) THEN
  ANTS_TAC THENL [ASM_REWRITE_TAC[] THEN ASM_MESON_TAC[]; ALL_TAC] THEN
  DISCH_THEN(MP_TAC o SPEC `c:real^N->bool`) THEN ASM_REWRITE_TAC[] THEN
  ASM_CASES_TAC `c:real^N->bool = {}` THEN
  ASM_REWRITE_TAC[POLYHEDRON_EMPTY] THEN
  ASM_CASES_TAC `c:real^N->bool = s` THEN ASM_REWRITE_TAC[] THEN
  DISCH_THEN SUBST1_TAC THEN MATCH_MP_TAC POLYHEDRON_INTERS THEN
  REWRITE_TAC[FORALL_IN_GSPEC] THEN
  ONCE_REWRITE_TAC[SIMPLE_IMAGE_GEN] THEN
  ASM_SIMP_TAC[FINITE_IMAGE; FINITE_RESTRICT] THEN
  REPEAT STRIP_TAC THEN REWRITE_TAC[IMAGE_ID] THEN
  MATCH_MP_TAC POLYHEDRON_INTER THEN
  ASM_REWRITE_TAC[POLYHEDRON_HYPERPLANE]);;
```

FACE_OF_POLYHEDRON_POLYHEDRON

```
polyhedron s /\ c face_of s ==> polyhedron c
```

HOL Light proof: could not be re-played by ATPs.

Alternative proof found by a hammer based on `FACE_OF_STILLCONVEX`:
Face *t* of a convex set *s* is equal to the intersection of *s* with the affine hull of *t*.

```
FACE_OF_STILLCONVEX:
 !s t:real^N->bool. convex s ==>
 (t face_of s <=>
  t SUBSET s /\ convex(s DIFF t) /\ t = (affine hull t) INTER s)
POLYHEDRON_IMP_CONVEX:
 !s:real^N->bool. polyhedron s ==> convex s
POLYHEDRON_INTER:
 !s t:real^N->bool. polyhedron s /\ polyhedron t
   ==> polyhedron (s INTER t)
POLYHEDRON_AFFINE_HULL:
 !s. polyhedron(affine hull s)
```

Low-level: Statistical Guidance of Connection Tableau

- learn guidance of every clausal inference in connection tableau (leanCoP)
- set of first-order clauses, *extension* and *reduction* steps
- proof finished when all branches are closed
- a lot of nondeterminism, requires backtracking
- *Iterative deepening* used in leanCoP to ensure completeness
- good for learning – the tableau compactly represents the proof state

Clauses:

$c_1 : P(x)$

$c_2 : R(x, y) \vee \neg P(x) \vee Q(y)$

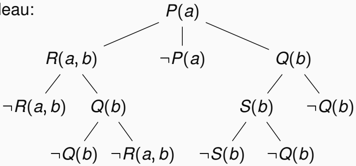
$c_3 : S(x) \vee \neg Q(b)$

$c_4 : \neg S(x) \vee \neg Q(x)$

$c_5 : \neg Q(x) \vee \neg R(a, x)$

$c_6 : \neg R(a, x) \vee Q(x)$

Closed Connection Tableau:



Statistical Guidance of Connection Tableau

- **MaLeCoP** (2011): first prototype Machine Learning Connection Prover
- extension rules chosen by naive Bayes trained on good decisions
- training examples: tableau features plus the name of the chosen clause
- initially slow: off-the-shelf learner 1000 times slower than raw leanCoP
- 20-time search shortening on the MPTP Challenge
- second version: 2015, with C. Kaliszyk
- both prover and naive Bayes in OCAML, fast indexing
- Fairly Efficient MaLeCoP = **FEMaLeCoP**
- 15% improvement over untrained leanCoP on the MPTP2078 problems
- using iterative deepening - enumerate shorter proofs before longer ones

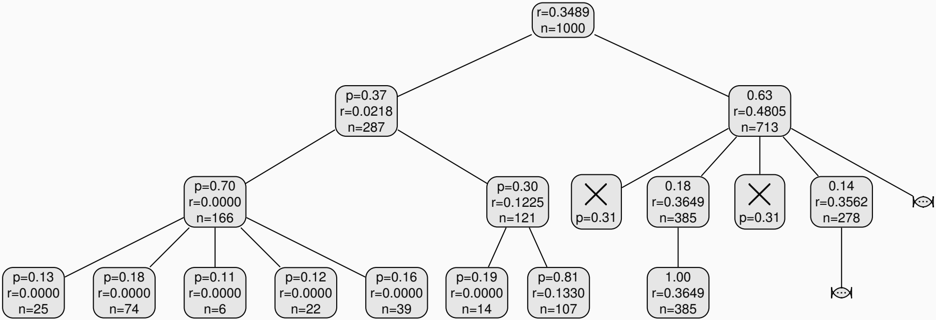
Statistical Guidance of Connection Tableau – rlCoP

- 2018: stronger learners via C interface to OCAML (boosted trees)
- remove iterative deepening, the prover can go arbitrarily deep
- added Monte-Carlo Tree Search (MCTS)
- MCTS search nodes are sequences of clause application
- a good heuristic to explore new vs exploit good nodes:

$$\frac{w_i}{n_i} + c \cdot p_i \cdot \sqrt{\frac{\ln N}{n_i}} \qquad \text{(UCT - Kocsis, Szepesvari 2006)}$$

- learning both *policy* (clause selection) and *value* (state evaluation)
- clauses represented not by names but also by features (generalize!)
- binary learning setting used: | proof state | clause features |
- mostly term walks of length 3 (trigrams), hashed into small integers
- many iterations of proving and learning

Tree Example



Statistical Guidance of Connection Tableau – rlCoP

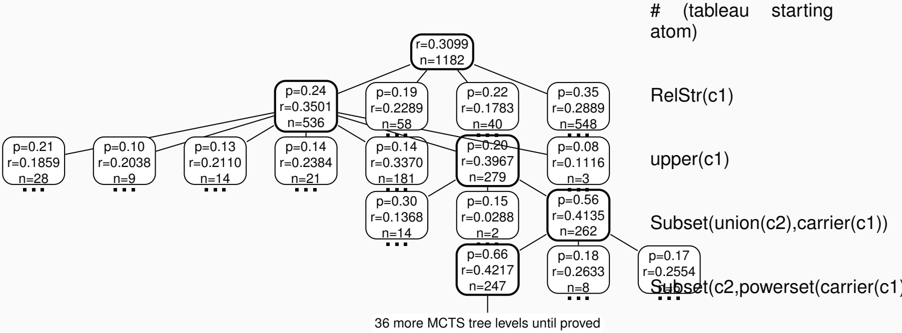
- On 32k Mizar40 problems using 200k inference limit
- nonlearning CoPs:

System	leanCoP	bare prover	rlCoP no policy/value (UCT only)
Training problems proved	10438	4184	7348
Testing problems proved	**1143**	431	804
Total problems proved	11581	4615	8152

- rlCoP with policy/value after 5 proving/learning iters on the training data
- $1624/1143 = 42.1$% improvement over leanCoP on the testing problems

Iteration	1	2	3	4	5	6	7	8
Training proved	12325	13749	14155	14363	14403	14431	14342	**14498**
Testing proved	1354	1519	1566	1595	**1624**	1586	1582	1591

More trees



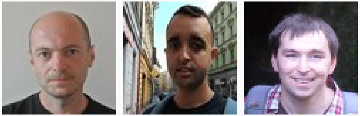
# (tableau starting atom)
RelStr(c1)
upper(c1)
Subset(union(c2),carrier(c1))
Subset(c2,powerset(carrier(c1)
r=0.3099 n=1182
p=0.24 r=0.3501 n=536
p=0.19 r=0.2289 n=58
p=0.22 r=0.1783 n=40
p=0.35 r=0.2889 n=548
p=0.21 r=0.1859 n=28
p=0.10 r=0.2038 n=9
p=0.13 r=0.2110 n=14
p=0.14 r=0.2384 n=21
p=0.14 r=0.3370 n=181
p=0.20 r=0.3967 n=279
p=0.08 r=0.1116 n=3
p=0.30 r=0.1368 n=14
p=0.15 r=0.0288 n=2
p=0.56 r=0.4135 n=262
p=0.66 r=0.4217 n=247
p=0.18 r=0.2633 n=8
p=0.17 r=0.2554
36 more MCTS tree levels until proved

Recent CoP Mutants: FLoP, GNN, RNN, lazyCoP

- FLoP – Finding Longer Proofs (Zombori et al, 2019)
- Curriculum Learning used for connection tableau over Robinson Arithmetic
- addition and multiplication learned perfectly from $1 * 1 = 1$
- headed towards learning algorithms/decision procedures from math data
- currently black-box, combinations with symbolic methods (ILP) our next target
- Using RNNs for better tableau encoding, prediction of actions ...
- ... even guessing (decoding) next tableau literals (Piotrowski 2020)
- plCoP (Zombori 20), GNN-CoP (Olsak 20), lazyCoP (Rawson) ...
- Zombori: learning new explainable Prolog actions (tactics) from proofs

ENIGMA (2017): Guiding the Best ATPs like E Prover

- ENIGMA (Jan Jakubuv, Zar Goertzel, Karel Chvalovsky, others)



- The proof state are two large heaps of clauses *processed/unprocessed*
- learn on E's proof search traces, put classifier in E
- positive examples: clauses (lemmas) used in the proof
- negative examples: clauses (lemmas) not used in the proof
- 2021 multi-phase architecture (combination of different methods):
  - fast gradient-boosted decision trees (GBDTs) used in 2 ways
  - fast logic-aware graph neural network (GNN - Olsak) run on a GPU server
  - logic-based subsumption using fast indexing (discrimination trees - Schulz)
- The GNN scores many clauses (context/query) together in a large graph
- Sparse - vastly more efficient than transformers ($\sim$100k symbols)
- 2021: leapfrogging and Split&Merge:
- aiming at learning reasoning/algo components

Feedback prove/learn loop for ENIGMA on Mizar data

- Done on 57880 Mizar problems recently
- Serious ML-guidance breakthrough applied to the best ATPs
- Ultimately a 70% improvement over the original strategy in 2019
- From 14933 proofs to 25397 proofs (all 10s CPU - no cheating)
- Went up to 40k in more iterations and 60s time in 2020
- 75% of the Mizar corpus reached in July 2021 - higher times and many runs: `https://github.com/ai4reason/ATP_Proofs`

	$\mathcal{S}$	$\mathcal{S} \odot \mathcal{M}_9^0$	$\mathcal{S} \oplus \mathcal{M}_9^0$	$\mathcal{S} \odot \mathcal{M}_9^1$	$\mathcal{S} \oplus \mathcal{M}_9^1$	$\mathcal{S} \odot \mathcal{M}_9^2$	$\mathcal{S} \oplus \mathcal{M}_9^2$	$\mathcal{S} \odot \mathcal{M}_9^3$	$\mathcal{S} \oplus \mathcal{M}_9^3$
solved	**14933**	16574	20366	21564	22839	22413	23467	22910	23753
$\mathcal{S}\%$	+0%	+10.5%	+35.8%	+43.8%	+52.3%	+49.4%	+56.5%	+52.8%	+58.4
$\mathcal{S}+$	+0	+4364	+6215	+7774	+8414	+8407	+8964	+8822	+9274
$\mathcal{S}-$	-0	-2723	-782	-1143	-508	-927	-430	-845	-454

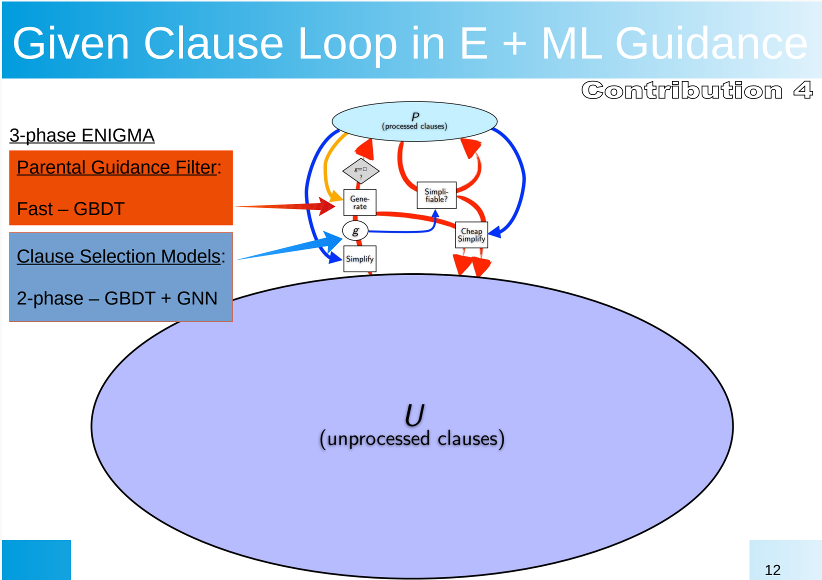
	$\mathcal{S} \odot \mathcal{M}_{12}^3$	$\mathcal{S} \oplus \mathcal{M}_{12}^3$	$\mathcal{S} \odot \mathcal{M}_{16}^3$	$\mathcal{S} \oplus \mathcal{M}_{16}^3$
solved	24159	24701	25100	**25397**
$\mathcal{S}\%$	+61.1%	+64.8%	+68.0%	**+70.0%**
$\mathcal{S}+$	+9761	+10063	+10476	+10647
$\mathcal{S}-$	-535	-295	-309	-183

ENIGMA Anonymous: Learning from patterns only

- The GNN and GBDTs only learn from formula structure, not symbols
- Not from symbols like $+$ and $*$ as Transformer & Co.
- E.g., learning on additive groups thus transfers to multiplicative groups
- Evaluation of old-Mizar ENIGMA on 242 new Mizar articles
- 13370 new theorems, $>$ 50% of them with new terminology:
- The 3-phase ENIGMA is **58%** better on them than unguided E
- While **53.5%** on the old Mizar (where this ENIGMA was trained)
- Generalizing, analogizing and transfer abilities unusual in the large transformer models
- Recently also trained on 300k Isabelle/AFP problems (Sledgehammer)

3-phase Anonymous ENIGMA

The 3-phase ENIGMA (single strategy) solves in 30s 56.4% of Mizar (bushy)



Neural Clause Selection in Vampire (M. Suda)

Deepire: Similar to ENIGMA:

- build a *classifier* for recognizing *good* clauses
- *good* are those that appeared in past proofs

Deepire's contributions:

- Learn from clause *derivation trees only*
  Not looking at what it says, just who its ancestors were.
- Integrate using *layered clause queues*
  A smooth improvement of the base clause selection strategy.
- Tree Neural Networks: constant work per derived clause
- A signature agnostic approach
- Delayed evaluation trick (not all derived need to be evaluated)

Preliminary Evaluation on Mizar "57880"

- Learn from 63595 proofs of 23071 problems (three 30s runs)
- Deepire solves 26217 (i.e. +4054) problems in a *single 10s run*

TacticToe: mid-level ITP Guidance (Gauthier'17,18)

- TTT learns from human and its own tactical HOL4 proofs
- No translation or reconstruction needed - native tactical proofs
- Fully integrated with HOL4 and easy to use
- Similar to rlCoP: policy/value learning for applying tactics in a state
- However much more technically challenging - a real breakthrough:
  - tactic and goal state recording
  - tactic argument abstraction
  - absolutization of tactic names
  - nontrivial evaluation issues
  - these issues have often more impact than adding better learners
- policy: which tactic/parameters to choose for a current goal?
- value: how likely is this proof state succeed?
- 66% of HOL4 toplevel proofs in 60s (better than a hammer!)
- similar recent work for Isabelle (Nagashima 2018), HOL Light (Google)

Tactician: Tactical Guidance for Coq (Blaauwbroek'20)

- Tactical guidance of Coq proofs
- Technically very challenging to do right - the Coq internals again nontrivial
- 39.3% on the Coq standard library, 56.7% in a union with CoqHammer (orthogonal)
- Fast approximate hashing for k-NN makes a lot of difference
- Fast re-learning more important than "cooler"/slower learners
- Fully integrated with Coq, should work for any development
- User friendly, installation friendly, integration friendly and maintenance friendly
- Took several years, but could become a very common tool for Coq formalizers

More on Conjecturing in Mathematics

- Targeted: generate intermediate lemmas (cuts) for a harder conjecture
- Unrestricted (theory exploration):
- Creation of interesting conjectures based on the previous theory
- One of the most interesting activities mathematicians do (how?)
- Higher-level AI/reasoning task - can we learn it?
- If so, we have solved math:
- ... just (recursively) divide Fermat into many subtasks ...
- ... and conquer (I mean: hammer) them away

Conjecturing and Proof Synthesis by Neural Language models

- Karpathy’15 - RNN experiments with generating fake Math over Stacks
- I have tried to use that for formal math in 2016 but it looked weak
- GPT (-2,3) looks stronger
- Renewed experiments in 2020 on:
- All Mizar articles, stripped of comments and concatenated together (78M)
- Articles with added context/disambiguation (156M) (types, names, thesis)
- TPTP proofs of 28271 Mizar/MPTP theorems by E/ENIGMA (658M)
- Just the conjecture and premises needed for the 28271 proofs printed in prefix notation
- Quite interesting results, server for Mizar authors
- Quickly taken up by others on HOL, Isabelle, MetaMath ...

Can you find the flaw(s) in this fake GPT-2 proof?

```
:: generated theorem with "proof"
theorem Th23: :: STIRL2_1:23
for X, Y being finite set st not X is empty & X c= Y
& card X = card Y holds X = Y
proof
  let X, Y be finite set ;
:: thesis: not X is empty & X c= Y & card X = card Y implies X = Y
  assume that
  A1: not X is empty and A2: X c= Y and A3: card X = card Y ;
:: thesis: X = Y
  card (Y \ X) = (card Y) - (card X) by A1, A3, CARD_2:44;
  then A4: card (Y \ X) = ((card Y) - 1) - (card X) by CARD_1:30;
  X = Y \ X by A2, A3, Th22;
  hence X = Y by A4, XBOOLE_0:def_10;
:: thesis: verum
end;
```

-:--- **card_tst.miz** 99% L2131 (Mizar Errors:13 hs Undo-Tree)

Figure: Fake full declarative GPT-2 “proof” - typechecks!

A correct conjecture that was too hard to prove

- Kinyon and Stanovsky (algebraists) confirmed that this cut is valid:

```
theorem Th10: :: GROUPP_1:10
for G being finite Group for N being normal Subgroup of G st
N is Subgroup of center G & G ./. N is cyclic holds G is commutative

The generalization that avoids finiteness:

for G being Group for N being normal Subgroup of G st
N is Subgroup of center G & G ./. N is cyclic holds G is commutative
```

More cuts

- In total 33100 in this experiment
- Ca 9k proved by trained ENIGMA
- Some are clearly false, yet quite natural to ask:

```
theorem :: SINCOS10:17

sec is increasing on [0, pi/2)
```

leads to conjecturing the following:

```
Every differentiable function is increasing.
```

QSynt: Semantics-Aware Synthesis of Math Objects

- Gauthier'19-22
- Synthesize math expressions based on semantic characterizations
- i.e., not just on the syntactic descriptions (e.g. proof situations)
- Tree Neural Nets and RL (MCTS, policy/value), used for:
- Guiding synthesis of a diophantine equation characterizing a given set
- Guiding synthesis of combinators for a given lambda expression
- 2022: invention of programs for OEIS sequences from scratch
- 50k sequences discovered so far:

  `https://www.youtube.com/watch?v=24oejR9wsXs`,

  `http://grid01.ciirc.cvut.cz/~thibault/qsynt.html`
- Many conjectures invented: 4 different characterizations of primes
- Non-neural (Turing complete) computing and semantics collaborates with the statistical learning

QSynt: synthesizing the programs/expressions

- Inductively defined set P of our *programs and subprograms*,
- and an auxiliary set F of binary functions (higher-order arguments)
- are the smallest sets such that $0, 1, 2, x, y \in P$, and if $a, b, c \in P$ and $f, g \in F$ then:

$$a + b, a - b, a \times b, a \; div \; b, a \; mod \; b, cond(a, b, c) \in P$$
$$\lambda(x, y).a \in F, \; loop(f, a, b), loop2(f, g, a, b, c), compr(f, a) \in P$$

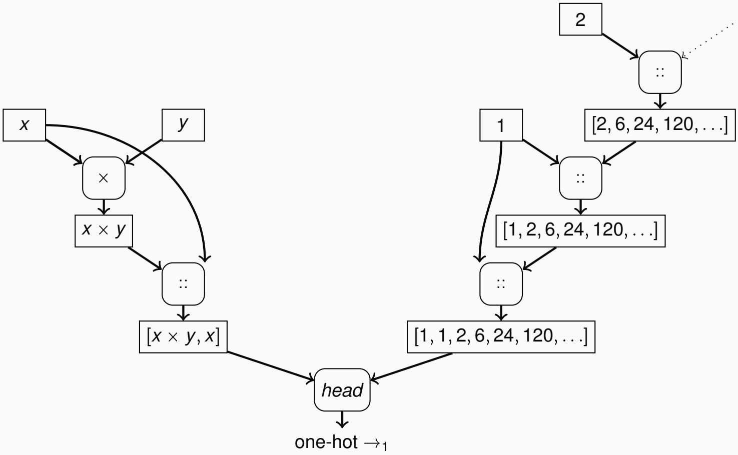
- Programs are built in reverse polish notation
- Start from an empty stack
- Use ML to repeatedly choose the next operator to push on top of a stack
- Example: Factorial is $loop(\lambda(x, y).\, x \times y, x, 1)$, built by:

$$[\,] \rightarrow_x [x] \rightarrow_y [x, y] \rightarrow_\times [x \times y] \rightarrow_x [x \times y, x]$$

$$\rightarrow_1 [x \times y, x, 1] \rightarrow_{loop} [loop(\lambda(x, y).\, x \times y, x, 1)]$$

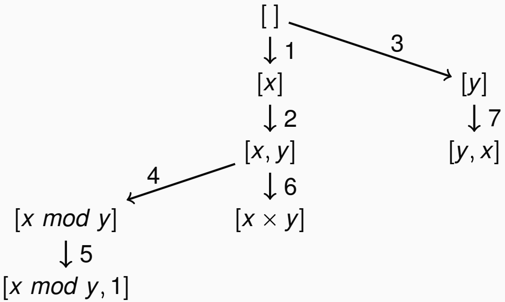
QSynt: Training of the Neural Net Guiding the Search

- The triple $((\mathit{head}([x \times y, x], [1, 1, 2, 6, 24, 120 \ldots]), \rightarrow_1)$ is a training example extracted from the program for factorial $\mathit{loop}(\lambda(x, y).\ x \times y, x, 1)$
- $\rightarrow_1$ is the action (adding 1 to the stack) required on $[x \times y, x]$ to progress towards the construction of $\mathit{loop}(\lambda(x, y).\ x \times y, x, 1)$.

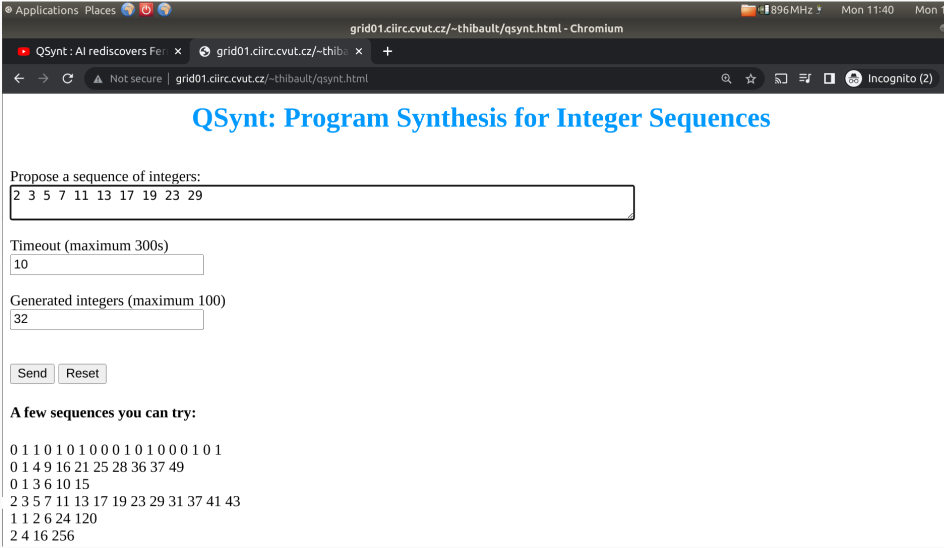


QSynt program search - Monte Carlo search tree

7 iterations of the search loop gradually extending the search tree. The set of the synthesized programs after the 7th iteration is $\{1, x, y, x \times y, x\ mod\ y\}$.



QSynt web interface for program invention



QSynt inventing Fermat pseudoprimes

Positive integers k such that $2^k \equiv 2 \mod k$. ($341 = 11 * 31$ is the first non-prime)

```
First 16 generated numbers (f(0),f(1),f(2),...):
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53
Generated sequence matches best with: A15919(1-75), A100726(0-59), A40(0-58)

Program found in 5.81 seconds
f(x) := 2 + compr(\x.loop(\(x,i).2*x + 2, x, 2) mod (x + 2), x)
Run the equivalent Python program here or in the window below:
```

BRYTHON

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English

Brython version: 3.10.6

run Python Javascript Share code

```python
def f2(X):
  x = 2
  for i in range (1,X + 1):
    x = 2*x + 2
  return x

def f1(X):
  x,i = 0,0
  while i <= X:
    if f2(x) % (x + 2) <= 0:
      i = i + 1
    x = x + 1
  return x - 1

def f0(X):
  return 2 + f1(X)

for x in range(32):
  print (f0(x))
```

```
2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
53
59
61
67
```

Lucas/Fibonacci characterization of (pseudo)primes

```
input sequence: 2,3,5,7,11,13,17,19,23,29

invented output program:
f(x) := compr(\(x,y).(loop2(\(x,y).x + y, \(x,y).x, x, 1, 2) - 1)
              mod (1 + x), x + 1) + 1

human conjecture: x is prime iff? x divides (Lucas(x) - 1)

PARI program:
? lucas(n) = fibonacci(n+1)+fibonacci(n-1)
? b(n) = (lucas(n) - 1) % n

Counterexamples (Bruckman-Lucas pseudoprimes):
? for(n=1,4000,if(b(n)==0,if(isprime(n),0,print(n))))
1
705
2465
2737
3745
```

QSynt inventing primes using Wilson's theorem

n is prime iff $(n-1)!+1$ is divisible by n (i.e.: $(n-1)! \equiv -1 \mod n$)

```
First 32 generated numbers (f(0),f(1),f(2),...):
0 1 1 0 1 0 1 0 0 0 1 0 1 0 0 0 1 0 1 0 0 0 1 0 0 0 0 0 1 0 1 0
Generated sequence matches best with: A10051(0-100), A252233(0-29), A283991(0-24)

Program found in 5.17 seconds
f(x) := (loop(\(x,i).x * i, x, x) mod (x + 1)) mod 2
Run the equivalent Python program here or in the window below:
```

Brython

Tutorial Demo Documentation Console Editor Gallery Resources

English

Brython version: 3.10.6

run Python Javascript Share code

```python
def f1(X):
  x = X
  for i in range (1,X + 1):
    x = x * i
  return x

def f0(X):
  return (f1(X) % (X + 1)) % 2

for x in range(32):
  print (f0(x))

```

```
0
1
1
0
1
0
1
0
0
0
1
0
```

Are two QSynt programs equivalent?

- As with primes, we often find many programs for one OEIS sequence
- It may be quite hard to see that the programs are equivalent
- A simple example for $0, 2, 4, 6, 8, \ldots$ with two programs f and g:
  - $f(0) = 0, f(n) = 2 + f(n-1)$ if $n > 0$
  - $g(n) = 2 * n$
  - conjecture: $\forall n \in \mathbb{N}. g(n) = f(n)$
- We can ask mathematicians, but we have thousands of such problems
- Or we can try to ask our ATPs (and thus create a large ATP benchmark)!
- Here is one SMT encoding by Mikolas Janota:

```
(set-logic UFLIA)
(define-fun-rec f ((x Int)) Int (ite (<= x 0) 0 (+ 2 (f (- x 1)))))
(assert (exists ((c Int)) (and (> c 0) (not (= (f c) (* 2 c))))))
(check-sat)
```

Inductive proof by Vampire of the $f = g$ equivalence

```
% SZS output start Proof for rec2
1. f(X0) = $ite($lesseq(X0,0), 0,$sum(2,f($difference(X0,1)))) [input]
2. ? [X0 : $int] : ($greater(X0,0) & ~f(X0) = $product(2,X0)) [input]
[...]
43. ~$less(0,X0) | iG0(X0) = $sum(2,iG0($sum(X0,-1))) [evaluation 40]
44. (! [X0 : $int] : (($product(2,X0) = iG0(X0) & ~$less(X0,0)) => $product(2,$sum(X0,1)) = iG0($sum(X0,1)))
    &  $product(2,0) = iG0(0))   => ! [X1 : $int] : ($less(0,X1) => $product(2,X1) = iG0(X1)) [induction hypo]
[...]
49. $product(2,0) != iG0(0) | $product(2,$sum(sK3,1)) != iG0($sum(sK3,1)) | ~$less(0,sK1) [resolution 48,41]
50. $product(2,0) != iG0(0) | $product(2,sK3) = iG0(sK3) | ~$less(0,sK1) [resolution 47,41]
51. $product(2,0) != iG0(0) | ~$less(sK3,0) | ~$less(0,sK1) [resolution 46,41]
52. 0 != iG0(0) | $product(2,$sum(sK3,1)) != iG0($sum(sK3,1)) | ~$less(0,sK1) [evaluation 49]
53. 0 != iG0(0) | $product(2,sK3) = iG0(sK3) | ~$less(0,sK1) [evaluation 50]
54. 0 != iG0(0) | ~$less(sK3,0) | ~$less(0,sK1) [evaluation 51]
55. 0 != iG0(0) | ~$less(sK3,0) [subsumption resolution 54,39]
57. 1 <=> $less(sK3,0) [avatar definition]
59. ~$less(sK3,0) <- (~1) [avatar component clause 57]
61. 2 <=> 0 = iG0(0) [avatar definition]
64. ~1 | ~2 [avatar split clause 55,61,57]
65. 0 != iG0(0) | $product(2,sK3) = iG0(sK3) [subsumption resolution 53,39]
67. 3 <=> $product(2,sK3) = iG0(sK3) [avatar definition]
69. $product(2,sK3) = iG0(sK3) <- (3) [avatar component clause 67]
70. 3 | ~2 [avatar split clause 65,61,67]
71. 0 != iG0(0) | $product(2,$sum(sK3,1)) != iG0($sum(sK3,1)) [subsumption resolution 52,39]
72. $product(2,$sum(1,sK3)) != iG0($sum(1,sK3)) | 0 != iG0(0) [forward demodulation 71,5]
74. 4 <=> $product(2,$sum(1,sK3)) = iG0($sum(1,sK3)) [avatar definition]
76. $product(2,$sum(1,sK3)) != iG0($sum(1,sK3)) <- (~4) [avatar component clause 74]
77. ~2 | ~4 [avatar split clause 72,74,61]
82. 0 = iG0(0) [resolution 36,10]
85. 2 [avatar split clause 82,61]
246. iG0($sum(X1,1)) = $sum(2,iG0($sum($sum(X1,1),-1))) | $less(X1,0) [resolution 43,14]
251. $less(X1,0) | iG0($sum(X1,1)) = $sum(2,iG0(X1)) [evaluation 246]
[...]
1176. $false <- (~1, 3, ~4) [subsumption resolution 1175,1052]
1177. 1 | ~3 | 4 [avatar contradiction clause 1176]
1178. $false [avatar sat refutation 64,70,77,85,1177]
% SZS output end Proof for rec2
% Time elapsed: 0.016 s
```

Neural Autoformalization (Wang et al., 2018)

- generate ca 1M Latex/Mizar pairs based on Bancerek's work
- train neural seq-to-seq translation models (Luong – NMT)
- evaluate on about 100k examples
- many architectures tested, some work much better than others
- very important latest invention: *attention* in the seq-to-seq models
- more data very important for neural training – our biggest bottleneck (you can help!)
- Recent addition: unsupervised methods (Lample et all 2018) – no need for aligned data!

Neural Autoformalization data

Rendered LaTeX	If $X \subseteq Y \subseteq Z$, then $X \subseteq Z$.
Mizar	`X c= Y & Y c= Z implies X c= Z;`
Tokenized Mizar	`X c= Y & Y c= Z implies X c= Z ;`
LaTeX	`If $X \subseteq Y \subseteq Z$, then $X \subseteq Z$.`
Tokenized LaTeX	`If $ X \subseteq Y \subseteq Z $ , then $ X \subseteq Z $ .`

Neural Autoformalization results

Parameter	Final Test Perplexity	Final Test BLEU	Identical Statements (%)	Identical No-overlap (%)
128 Units	3.06	41.1	40121 (38.12%)	6458 (13.43%)
256 Units	1.59	64.2	63433 (60.27%)	19685 (40.92%)
512 Units	1.6	**67.9**	66361 (63.05%)	21506 (44.71%)
1024 Units	**1.51**	61.6	**69179 (65.73%)**	**22978 (47.77%)**
2048 Units	2.02	60	59637 (56.66%)	16284 (33.85%)

Neural Fun – Performance after Some Training

Rendered LaTeX	Suppose s_8 is convergent and s_7 is convergent . Then $\mathop{\rm lim}(s_8+s_7) = \mathop{\rm lim} s_8 + \mathop{\rm lim} s_7$
Input LaTeX	`Suppose $ { s _ { 8 } } $ is convergent and $ { s _ { 7 } } $ is convergent . Then $ \mathop { \rm lim } ( { s _ { 8 } } { + } { s _ { 7 } } } ) \mathrel { = } \mathop { \rm lim } { s _ { 8 } } { + } \mathop { \rm lim } { s _ { 7 } } $ .`
Correct	`seq1 is convergent & seq2 is convergent implies lim ( seq1 + seq2 ) = ( lim seq1 ) + ( lim seq2 ) ;`
Snapshot-1000	`x in dom f implies ( x * y ) * ( f | ( x | ( y | ( y | y ) ) ) ) = ( x | ( y | ( y | ( y | y ) ) ) ) ) ;`
Snapshot-2000	`seq is summable implies seq is summable ;`
Snapshot-3000	`seq is convergent & lim seq = 0c implies seq = seq ;`
Snapshot-4000	`seq is convergent & lim seq = lim seq implies seq1 + seq2 is convergent ;`
Snapshot-5000	`seq1 is convergent & lim seq2 = lim seq2 implies lim_inf seq1 = lim_inf seq2 ;`
Snapshot-6000	`seq is convergent & lim seq = lim seq implies seq1 + seq2 is convergent ;`
Snapshot-7000	`seq is convergent & seq9 is convergent implies lim ( seq + seq9 ) = ( lim seq ) + ( lim seq9 ) ;`

Unsupervised NMT Fun on Short Formulas

```
len <* a *> = 1 ;                   len <* a *> = 1 ;
assume i < len q ;                  i < len q ;
len <* q *> = 1 ;                   len <* q *> = 1 ;
s = apply ( v2 , v1 ast t ) ;       s = apply ( v2 , v1 ) . t ;
s . ( i + 1 ) = tt . ( i + 1 )      s . ( i + 1 ) = tau1 . ( i + 1 )
1 + j <= len v2 ;                   1 + j <= len v2 ;
1 + j + 0 <= len v2 + 1 ;           1 + j + 0 <= len v2 + 1 ;
let i be Nat ;                      i is_at_least_length_of p ;
assume v is_applicable_to t ;       not v is applicable ;
let t be type of T ;                t is_orientedpath_of v1 , v2 , T ;
a ast t in downarrow t ;            a *' in downarrow t ;
t9 in types a ;                     t `2 in types a ;
a ast t <= t ;                      a *' <= t ;
A is_applicable_to t ;              A is applicable ;
Carrier ( f ) c= B                  support ppf n c= B
u in B or u in { v } ;              u in B or u in { v } ;
F . w in w & F . w in I ;           F . w in F & F . w in I ;
GG . y in rng HH ;                  G0 . y in rng ( H1 ./. y ) ;
a * L = Z_ZeroLC ( V ) ;            a * L = ZeroLC ( V ) ;
not u in { v } ;                    u >> v ;
u <> v ;                            u <> v ;
v - w = v1 - w1 ;                   vw = v1 - w1 ;
v + w = v1 + w1 ;                   v + w = v1 + w1 ;
x in A & y in A ;                   assume [ x , y ] in A ;
```

Future: AITP Challenges/Bets from 2014

- 3 AITP bets from my 2014 talk at Institut Henri Poincare
  - In 20 years, 80% of Mizar and Flyspeck toplevel theorems will be provable automatically (same hardware, same libraries as in 2014 - about 40% then)
  - In 10 years: 60% (DONE already in 2021 - 3 years ahead of schedule)
  - In 25 years, 50% of the toplevel statements in LaTeX-written Msc-level math curriculum textbooks will be parsed automatically and with correct formal semantics (this may be faster than I expected)
- My (conservative?) estimate when we will do Fermat:
  - Human-assisted formalization: by 2050
  - Fully automated proof (hard to define precisely): by 2070
  - See the Foundation of Math thread: `https://bit.ly/300k9Pm`
- Big challenge: Learn complicated symbolic algorithms (not black box - motivates also our OEIS research)

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- Learning2Reason people at Radboud University Nijmegen:
  - Herman Geuvers, Tom Heskes, Daniel Kuehlwein, Evgeni Tsivtsivadze, ...
- Google Research: Christian Szegedy, Geoffrey Irving, Alex Alemi, Francois Chollet, Sarah Loos
- ... and many more ...
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Thanks and Advertisement

- Thanks for your attention!
- AITP – Artificial Intelligence and Theorem Proving
- September 4–9, 2022, Aussois, France, `aitp-conference.org`
- ATP/ITP/Math vs AI/Machine-Learning people, Computational linguists
- Discussion-oriented and experimental
- Grown to 80 people in 2019
- Will be hybrid in 2022 as in 2021 and 2020
- Invited talks by J. Araujo, K. Buzzard, J. Brandstetter, W. Dean and A. Naibo, M. Rawson, T. Ringer, S. Wolfram