Flexible Proof Production in an Industrial-Strength SMT Solver

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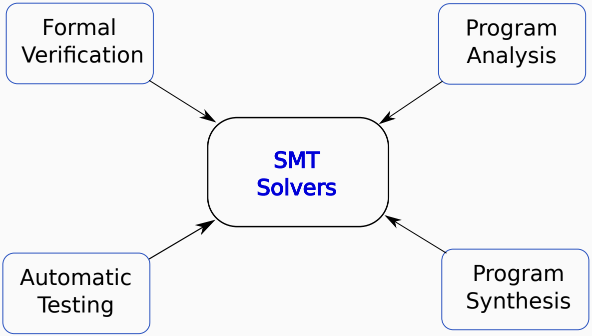
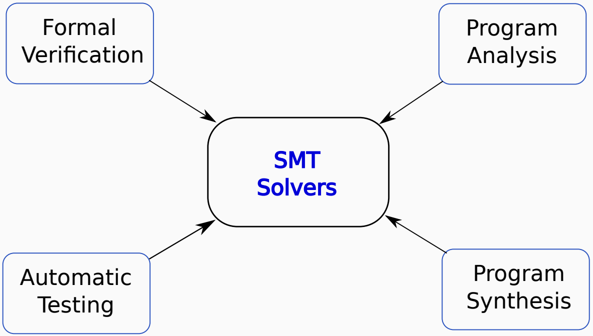
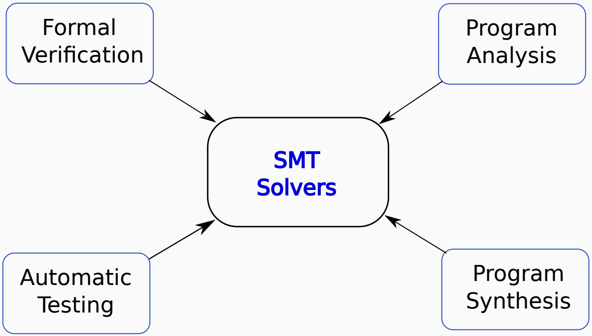
Special thanks: Vinícius Braga[1]

[1] Universidade Federal de Minas Gerais, [2] The University of Iowa, [3] Stanford University, [4] Bar-Ilan University


Formal Verification
Program Analysis
SMT Solvers
Automatic Testing
Program Synthesis



Called billions of times a day...



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- Even a tiny fraction of wrong answers is bad

Bugs in SMT Solvers

- State-of-the-art solvers are large projects:
  - Bitwuzla: 90k LoC (C/C++)
  - cvc5: 300k LoC (C++)
  - z3: 500k LoC (C++)

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- How do developers try to avoid bugs?
  - Code reviews
  - Testing on benchmark sets
  - Random input testing
- But bugs remain:
  - Every year SMT-COMP has disagreements between solvers
  - Fuzzing tools often find bugs in solvers

SMT Recap

// Input

Many-sorted, first-order logic formula:

$$\overbrace{\text{contains}(x, \texttt{"FLoC"})}^{\text{String constraint}} \overbrace{\land}^{\text{Boolean connective}} \overbrace{|x| \geq 5}^{\text{Integer constraint}}$$

- Does there exist a string x s.t. x contains $\texttt{"FLoC"}$ and is at least five characters long?

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// Examples of Theories

- Integer/real arithmetic: $5 + x \geq y$
- Bit-vectors: $\mathsf{bvule}(x, \texttt{0xFF})$
- Strings: $\mathsf{substr}(x, 0, 3) = \texttt{"foo"}$

Can We Just Certify the Solvers?

- Large, complex code bases are too costly to certify
- A (simpler) certified system can be too slow [FBL18; Fle19]
- Certifying/qualifying a system freezes it, potentially blocking improvements
  - Working around adding new features slow and costly [BD18]

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For satisfiable inputs: Evaluate formula on values of model generated by solver

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Model: $\mathcal{M} = \{x \mapsto \texttt{"FLoC-2022"}\}$

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What about unsatisfiable inputs?

Agenda

- Uses of proofs
- Challenges in producing proofs
- Making a solver proof producing:
  - Proofs through instrumentation: Primary approach in cvc5
  - Proofs through reconstruction: Detailed rewrite proofs in cvc5
- Current and future work

Proofs: A New Hope

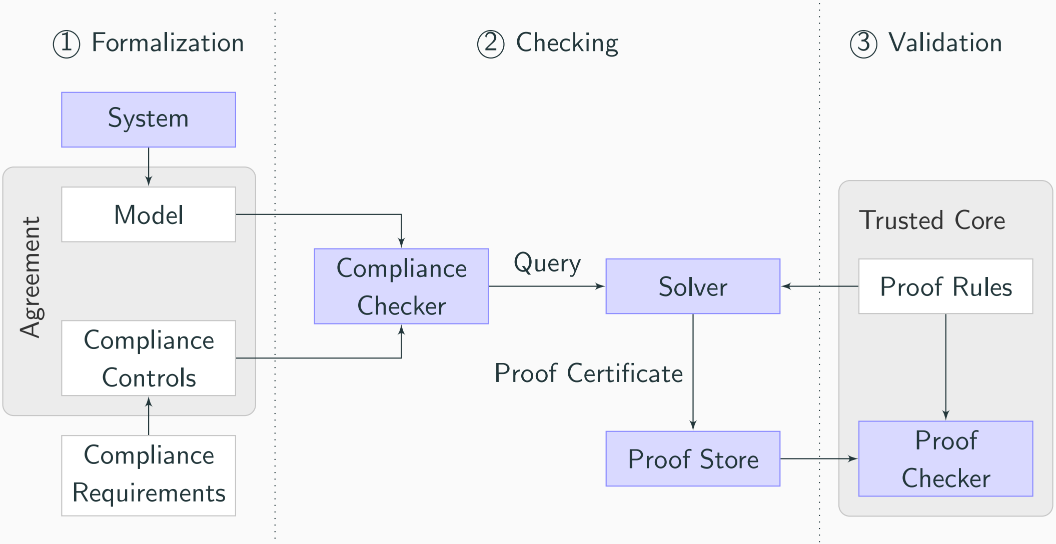
- Proofs are a justification of the logical reasoning the solver has performed to find a solution
- A proof can be checked *independently*
  - Smaller trusted base: LFSC 5.5k (C++) + 2k (signatures) LoC vs. CVC5 300k LoC
  - Proof checking is generally more efficiently than solving the problem
- Confidence in results is decoupled from the solver's implementation

Demo: A Simple Proof

Applications of SMT Proofs

- Strong correctness guarantees
    - High-quality proofs can be used to facilitate automated compliance
- Integrations with other systems
    - Automation in interactive theorem proving
    - External proof cecking can identify bugs in proof rules
- Valuable for debugging
- Formalization of proof rules improves code base
    - Uncovers existing issues
    - Forces modular and clean code design
    - Improves tool robustness
- A rich source of data that can be mined for various purposes (e.g., interpolation)

Applications of SMT Proofs: Compliance



Challenges for SMT proofs

- Collecting and storing proofs efficiently

  Many attempts, no silver bullet

  [SZS04; KBT+16; HBR+15; Mos08; MB08; Sch13; KV13; WDF+09; BODF09]
- Proofs for sophisticated preprocessing and rewriting techniques

  Initial progress but many challenges remain

  [BBFF20]
- Proofs for complex procedures in theory solvers (e.g., CAD, strings)

  Open
- Standardizing a proof format

  Open
- Scalable, trustworthy checking

  Many attempts, no silver bullet

  [BBP13; SOR+13; EMT+17; BBFF20; SFD21]

The Journey

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    - Every rule associated with an internal proof checker

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    - Proof reconstruction (elaboration) via internal post-processing
  - Support internal proof format and conversions to different proof formats
    - LFSC, Lean, Alethe

Proof module architecture for CDCL($\mathcal{T}$)

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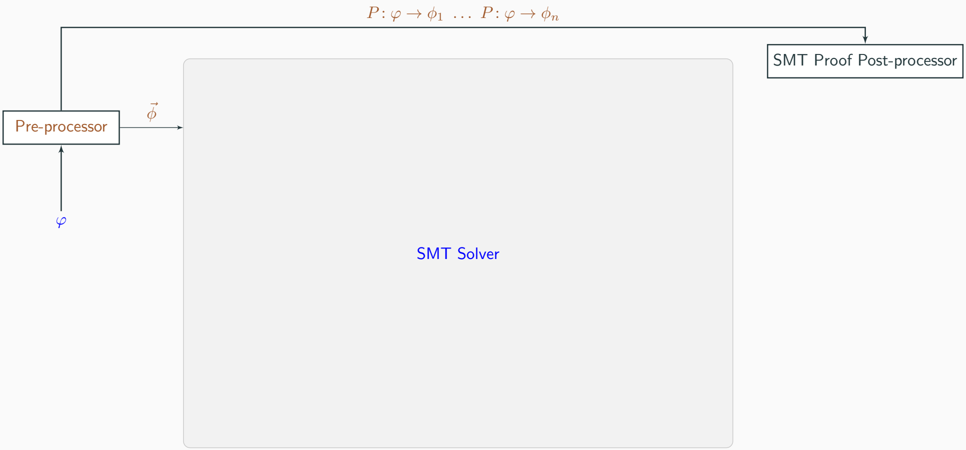
- Preprocessor simplifies formula globally:
  $x \simeq t \wedge F[x] \;\mapsto\; F[t] \qquad F[(ite\ P\ t_1\ t_2)] \;\mapsto\; F[t'] \wedge P \rightarrow t' \simeq t_1 \wedge \neg P \rightarrow t' \simeq t_2$

Proof module architecture for CDCL($\mathcal{T}$)



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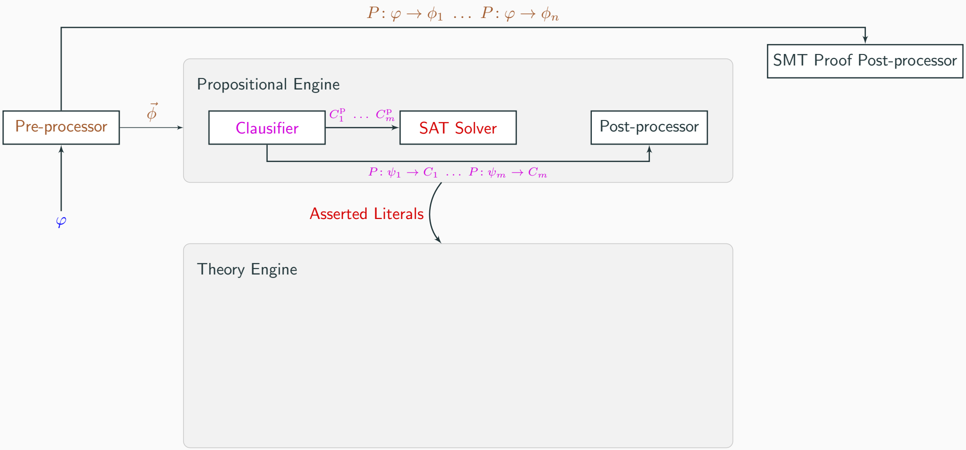
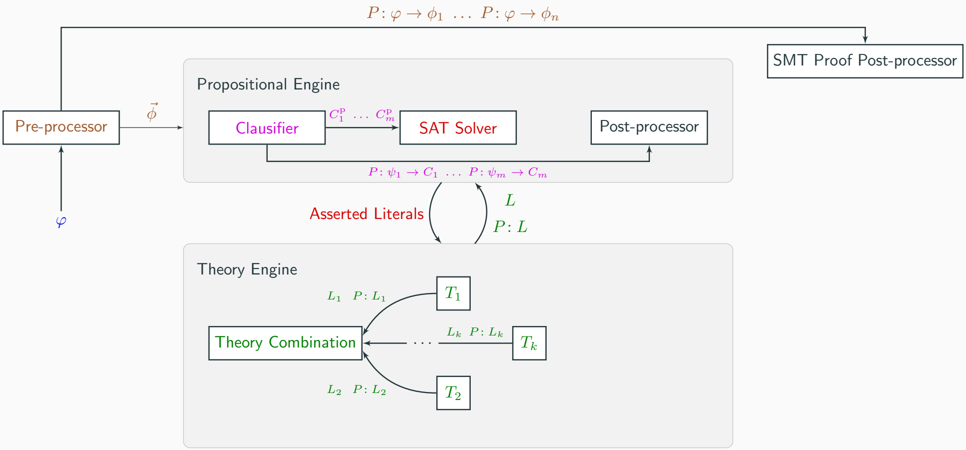


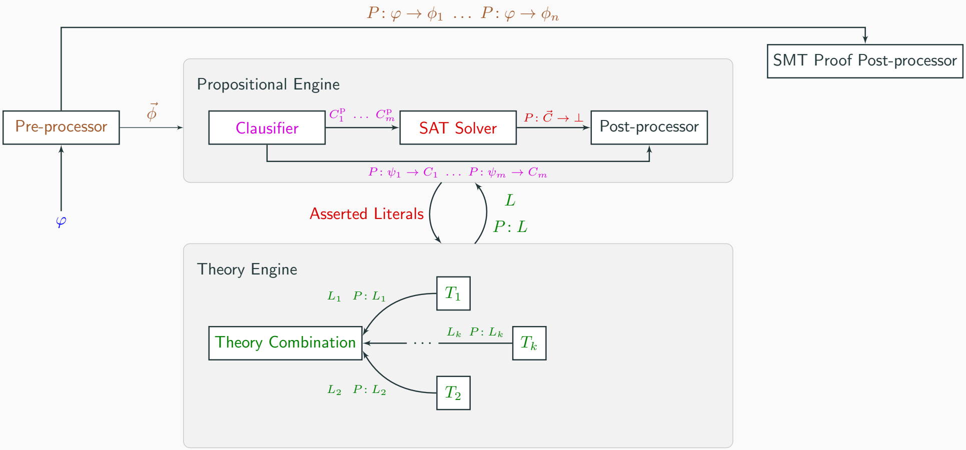
- Clausifier converts to Conjunctive Normal Form (CNF)
  SAT solver asserts literals that must hold based on Boolean abstraction

Proof module architecture for CDCL($\mathcal{T}$)



- Theory solvers check consistency in the theory

Proof module architecture for CDCL($\mathcal{T}$)



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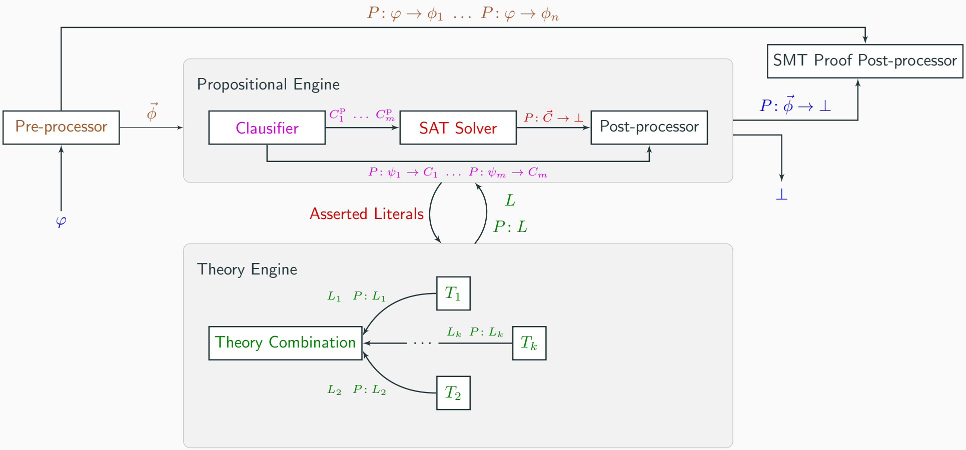

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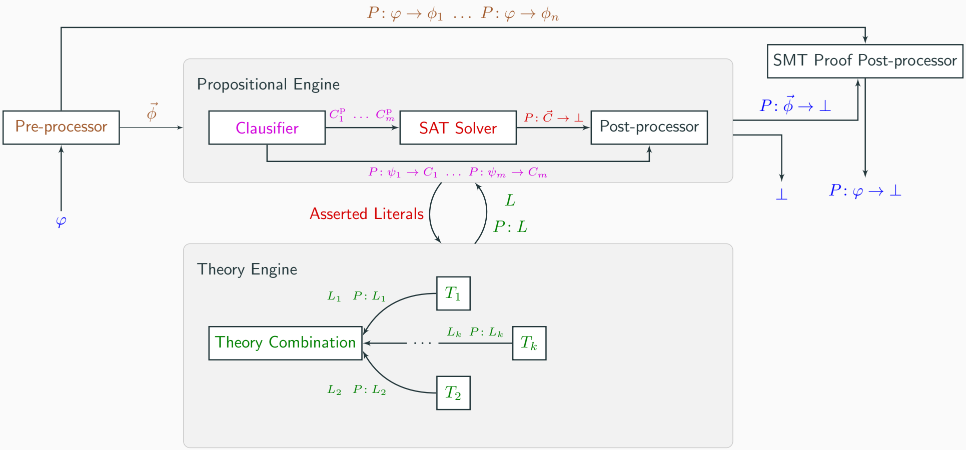

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Main components: Internal proof calculus

- Rules for equality reasoning (congruence closure)
- Rules for rewriting, substitution
  - Coarse-grained rules for capturing multiple core utilities
- Rules for witness forms
  - Enable introduction and correct handling of new symbols
- Rules for scoped reasoning
  - Enable local reasoning, via assumptions and $\Rightarrow$-introduction
- Theory-specific rules
  - Boolean (clausification, resolution, ...)
  - Arithmetic (linear, non-linear, integer, rationals, transcendentals)
  - Arrays, Datatypes, Bit-vectors, Quantifiers, ...

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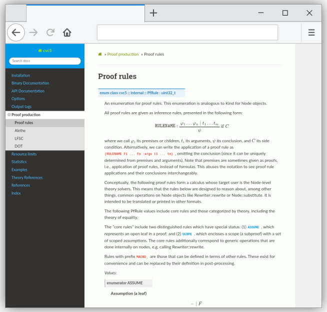
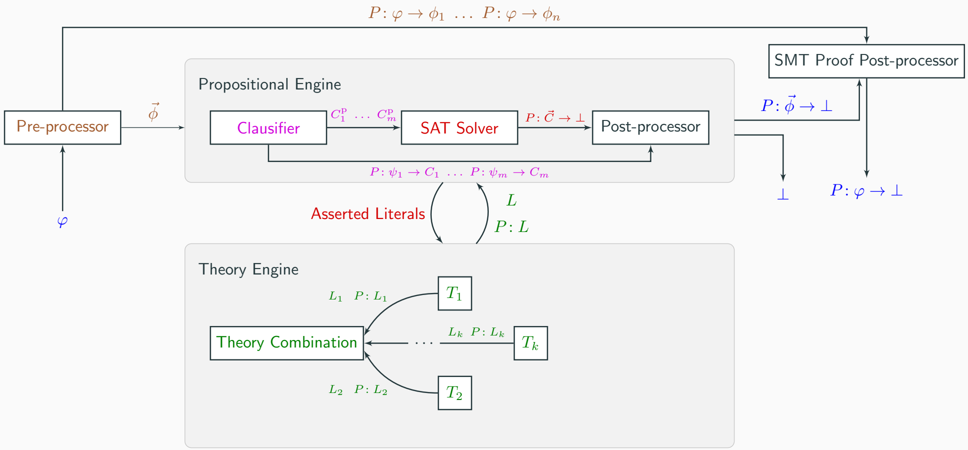


Documentation: `https://cvc5.github.io/docs/latest/proofs/proof_rules.html`

Main components: Library of proof generators

- Encapsulate common patterns for building proofs
- Solving components store information during solving
- Derived facts are distributed with associated proof generators
- When proof generator is requested for fact φ, its internal information is used to produce the proof $P : \varphi$.

Proof module architecture



- Actually, *proof generators* are transmitted between components
- Only at the post-processors are proofs requested (and fully computed)

Proof generation for substitution and rewriting

- Substitution and rewriting inferences recorded without further details
- No need to instrument utilities to track how terms are converted
  - Only macro steps and used rewrites rules are stored in generators

$$\dfrac{a \simeq 0 \qquad\qquad b \simeq 1}{(a > b \wedge F) \simeq \bot}\ \textsf{SR}$$

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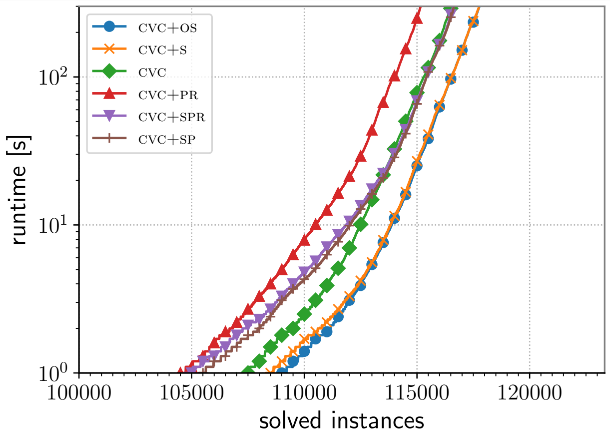
- Heavily used for strings, preprocessing, bitblasting, and so on.

Evaluation: proof production cost

- Techniques (currently) incompatible with proofs (O)
  - Variable and clause elimination (SAT solver), EUF symmetry breaking, off-the-shelf SAT solvers for BV bitblasted constraints
- Simplification under global assumptions (S)
- Producing proofs (P)
- Reconstructing fine-grained steps from coarse ones (R)

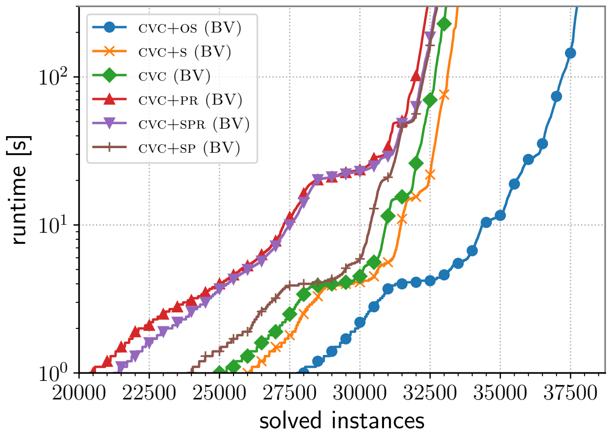
- Benchmarks
  - 123k NON-BVS benchmarks
  - 39k BVS benchmarks

Evaluation: non-BVs



- S is fundamental for performance
- P and R significant but not critical overhead
  - CVC+SP is in total 1.7× slower than CVC+S
  - CVC+SPR is in total 1.8× slower than CVC+S

Evaluation: BVs



- O is critical
- P and R significant but not critical overhead
  - CVC+SP is in total 2.6× slower than CVC+S
  - CVC+SPR is in total 3.9× slower than CVC+S

Evaluation: Proof Coverage

Perfect proofs are those without coarse-grained steps.

- 92% of perfect proofs in BVs
- 80% in non-BVs
  - Culprits are mostly yet-to-be-supported theory preprocessing passes
  - Also all non-linear arithmetic inferences from cylindric algebraic coverings
- 100% in QF_S, 80% in QF_SLIA

The Challenge with Rewrites

- Modern SMT solvers implement <u>hundreds</u> of rewriting rules for state-of-the-art performance
  - String solver in cvc5: Over 200 rules in $3{,}000$ lines of C++ code
  - Example:

$$\mathsf{substr}(\texttt{""}, m, n) \rightsquigarrow \texttt{""}$$

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- Many proof applications require detailed proofs
  - Easier proof checking, better integration with interactive theorem provers
  - Required: Individual proof rules for rewrite rules

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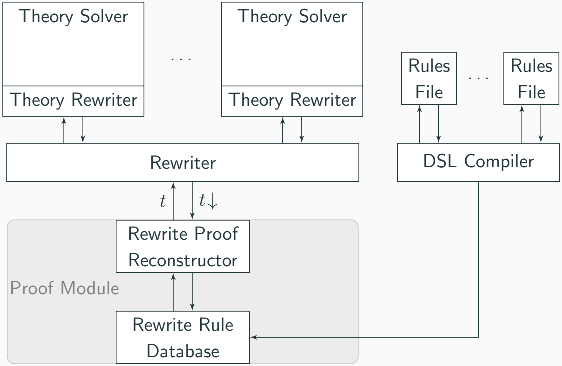
- Many proof applications require detailed proofs
  - Easier proof checking, better integration with interactive theorem provers
  - Required: Individual proof rules for rewrite rules
- Traditional approach: Instrumenting code
  - Difficult and tedious: Define proof rule and instrument code for every rewrite

Proofs for Rewrites: Our Approach



- Treat rewriter as black box and reconstruct proofs for rewrites externally
- A domain-specific language (DSL), RARE, to specify a database of rewrite rules
- A compiler for RARE that generates the C++ code that populates the rewrite rule database
- A general reconstruction algorithm, applied as a post-processor

Demo: Detailed Rewrite Proofs

Rare: Design Goals

- Succinct: Writing rewrite rules should be simple and concise.
- Expressive: Support for the majority of the rewrite rules in a state-of-the-art solver
- Accessible: Easy to parse and understand

Rare: Basic Rules

```
(define-rule substr-empty ((m Int) (n Int))
  (str.substr "" m n) "")
```

Rare: Matching

```
(define-rule eq-refl ((t ?)) (= t t) true)
```

Rare: Lists

```
(define-rule str-concat-flatten (
    (xs String :list) (s String)
    (ys String :list) (zs String :list))
  (str.++ xs (str.++ s ys) zs) ; match
  (str.++ xs s ys zs))         ; target
```

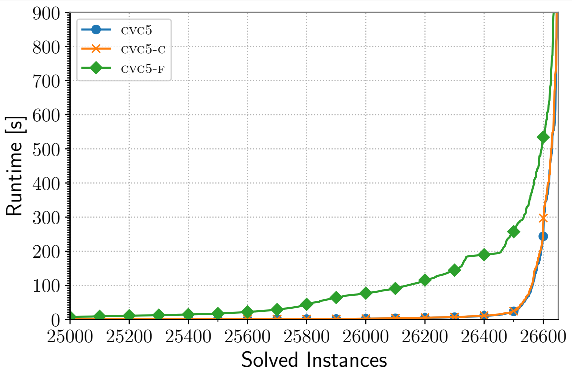
Rare: Conditional Rules

```
(define-cond-rule concat-clash (
    (s1 String) (s2 String :list)
    (t1 String) (t2 String :list))
  (and (= (str.len s1) (str.len t1)) ; precondition
    (not (= s1 t1)))
  (= (str.++ s1 s2) (str.++ t1 t2))  ; match
  false)                             ; target
```

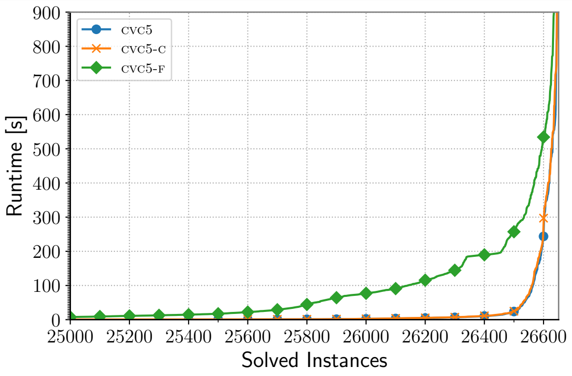
Rare: Fixed-Point Rules

```
(define-rule* str-len-concat-rec (
    (s1 String) (s2 String)
    (rest String :list))
  (str.len (str.++ s1 s2 rest)) ; match
  (str.len (str.++ s2 rest))     ; target
  (+ (str.len s1) _))            ; context
```

Rare: Evaluation



Rare: Evaluation



- Rewrites reconstructed: 95% for problems from the industrial set and of 87% for SMT-LIB
- Fully fine-grained: 20% of the proofs for industrial benchmarks, 23% of all proofs for SMT-LIB benchmarks with rewrite steps ($6{,}120$ out of $26{,}611$)

Current/Future Work

- Detailed proofs for remaining theories (e.g., non-linear arithmetic)
- Integration of DRAT proofs for propositional reasoning
- Integration with interactive theorem provers: Lean, Isabelle/HOL, Coq
- Proof components
  - Producing and checking fully detailed can be costly
  - Idea: Produce proofs that can be expanded <u>on-demand</u>
- More complete rules for rewriting
- Standardization of a proof format
  - Proof exhibition track at SMT-COMP 2022

Anecdotes

- Internal proof checker is highly valuable for development
- Error localization for proofs is important
- Formalization of proof rules uncovers existing issues
- Performance issues
  - In a few cases, proof checker indicated it could prove something stronger
- Soundness issues
  - Cannot write proper proof checker if the reasoning of the solver is wrong
- Proofs are also valuable for debugging
  - Soundness bug reported, proofs used to easily isolate the incorrect rewrite
- Combination of approaches for proof generation

Conclusion

- Proofs are integral for the trustworthiness SMT solvers (and have other applications)
- Fine-grained proofs are now available for most of cvc5's reasoning
  - Combination of instrumentation and reconstruction
  - Strings and simplification under global assumptions were special milestones
  - Detailed proofs for rewriting coming soon
- Multiple proof formats are supported
  - Integration into multiple proof checkers are ongoing
  - Formalization of new calculi in Lean, LFSC, Isabelle/HOL
  - DOT format and web-based proof visualizer

More information: `https://cvc5.github.io/`

SMT Proof Standardization Update today at 16:00

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How some proofs look like

$$\frac{A \vee \ell \qquad B \vee \bar{\ell}}{A \vee B}$$

$$\neg(a \simeq b) \vee f(a) \simeq f(b)$$
$$\neg(y > 1) \vee \neg(x < 1) \vee y > x$$

$$\frac{\varphi_1 \wedge \cdots \wedge \varphi_n}{\varphi_i}$$

$$\neg(\varphi_1 \wedge \cdots \wedge \varphi_n) \vee \varphi_i$$

A particular challenge has been String solving

- Preprocessing
- Clausification
- SAT solving
- UF theory solver
- Linear Arithmetic solver
- Theory combination
- Quantifier instantiation
- Rewriting
    - Including complex string methods [RNBT19]
- Strings theory solver
    - Core calculus [LRT+14]
    - Extended function reductions [RWB+17]
    - Regular expression unfolding

References

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